A method to compensate kinematic cross-talk

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***Abstract*— In this paper we want to show that it is possible to compensate the misalignment of a joint with respect to a fixed reference frame or the misalignment of a moving frame with respect to the joint when the joint angles vary periodically. In this case it is possible to calculate a correction matrix which strongly eliminates kinematic cross-talk. If both types of misalignment occur together, the cross-talk can be minimised by a least-squares optimisation.**

Keywords-component;kinematic cross-talk, axes misalignment

I. INTRODUCTION

It is well known that if the principal rotation axis, such as the flexion/extension axis of the knee is not perfectly aligned with the reference axis of an orthogonal frame, then the decomposition of the rotation matrix in X,Y and Z-components gives spurious components along the axes which are orthogonal to the main axis. ([1],[2]) This phenomen which is a concern for joints which articulate principally about one axis is also known as kinematic cross-talk ([3], [4]). A considerable alteration of the predicted screw-home motion for the knee with changes in the alignment of the joint axis was reported in [5]. The correction procedure which corrects the misaligment is described as reorientation. It was applied initially by [6], and the reorientation was based on the zeroing of the abduction/adduction and the internal/external rotation at the point of maximum knee flexion. The reorientation based on the concept of a mean flexion/extension axis obtained from the FHA, is described in [7]. The method was validated with the help of a special set-up modelling a perfect knee. In this paper we present a method for reorientation based on the use of classic rotation matrices. The determination of the FHA thus can be avoided and the rotation matrices obtained with several measurement devices can be used without differentiation. First we want to show by numeric simulation that for a motion around a principal axis, a nearly exact solution can easily be obtained if this principal axis has a misalignment with respect to the fixed frame or if there is an alignment uncertainty of the moving frame with respect to the joint only. Next, to compensate a misalignment between the pincipal axis and both the fixed and the moving frame, an optimisation technique based on the principle of least squares is described. The reoriented reference frame in this case is obtained by minimising the components induced by cross-talk.

II. SINGLE-SIDED MISALIGNMENT

A. The estimation of the amount of misalignment

To illustrate the method for compensating the misalignment, a matrix describing a main rotation angle δ and coupled components α_c, β_c is generated numerically with the expression

$$R(\alpha_c, \beta_c, \delta) = R_x(\alpha_c) R_y(\beta_c) R(\delta). \qquad (1)$$

The main rotation angle is a sinusoidal fluctuation around a mean

$$\delta = \delta_m + \delta_0 \sin(\omega_0 t). \qquad (2)$$

The coupled components are given by

$$\begin{aligned} \alpha_c &= \alpha_{c0} \sin(\omega_0 t) \\ \beta_c &= \beta_{c0} \sin(\omega_0 t) \end{aligned}. \qquad (3)$$

The joint rotation matrix in (1) is multiplied at the right side with a matrix $R(\varepsilon)$ to represent a moving frame which is misaligned with respect to the joint. This matrix, called misalignment matrix, is described by a finite rotation ε around a unit vector $\bar{n}$ and is generated with the Rodrigues expression

$$R(\varepsilon) = I + \sin(\varepsilon)\bar{n}^* + (1 - \cos(\varepsilon))\bar{n}^*\bar{n}^*. \qquad (4)$$

with the adjoint matrix $\bar{n}^*$ given by [6]

$$\bar{n}^* = \begin{bmatrix} 0 & -n_z & n_y \\ n_z & 0 & -n_x \\ -n_y & n_x & 0 \end{bmatrix}. \qquad (5)$$

Let the global rotation matrix R^0 , given by this product

$$R^0 = R(\alpha_c, \beta_c, \delta) R(\varepsilon). \qquad (6)$$

be decomposed in three successive rotations in the sequence XYZ, such that

$$R^0 = R_x(\alpha_0) R_y(\beta_0) R_z(\gamma_0). \qquad (7)$$

The angle around the Z-axis can be calculated with

$$\gamma_0 = -\mathrm{atan}\left(\frac{R^0_{0,1}}{R^0_{0,0}}\right). \qquad (8)$$

Let one period correspond to 360 samples corresponding to the integer k, and let the deviation angle be ε be -9° and the unit vector arbitrary given by

$$\bar{n} = [.1 \quad -.3 \quad .9486]. \qquad (9)$$

The misalignment corresponds to small rotations around all axis of the joint given by

$$R(\varepsilon) = R_z(-8.6°)R_y(2.6°)R_x(-1.1°). \qquad (10)$$

Due to the error of -8.6° along the Z axis the main rotation angle γ_0 with (8) is shifted by this amount with respect to the true angle δ in (2) as shown in Fig. 1

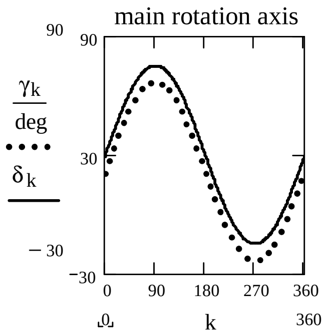

Figure 1. Given and calculated main-rotation

To calculate the misalignment matrix the mean value of this main rotation angle is calculated. Next the global matrix is multiplied from the left with a matrix which corresponds to an inverse rotation by this amount around the principal Z-axis. Assuming the coupled components α_c, β_c in (1) to be small, one finds with (6) that

$$R_z^{-1}(\gamma_m)R^0 \cong R_z^{-1}(\gamma_m)R_z(\delta)R(\varepsilon) = R_z(\delta - \gamma_m)R(\varepsilon). \qquad (11)$$

Consider the ZYX decomposition of the misalignment matrix

$$R(\varepsilon) = R_z(\varepsilon_z)R_y(\varepsilon_y)R_x(\varepsilon_x). \qquad (12)$$

then (11) becomes

$$R_z^{-1}(\gamma_m)R^0 \cong R_z(\delta - \gamma_m + \varepsilon_z)R_y(\varepsilon_y)R_x(\varepsilon_x). \qquad (13)$$

Obviously it is not possible to determine the misalignment angle ε_z. To calculate the misalignment angles $\varepsilon_x, \varepsilon_y$, the matrix obtained in (13) is decomposed into X and Y components only,

$$R_z^{-1}(\gamma_m)R^0 = R_y(\nu)R_x(\mu). \qquad (14)$$

By solving (14) for the variables μ, ν one obtains time-variable solutions, instead of constant values because the Z-component has been ignored in the decomposition. The average values of these angles however are an estimate for the misalignment angles $\varepsilon_x, \varepsilon_y$. This is a consequence of the fact that the principal angle δ is sinusoidal. When the main rotation angle reaches the value $(\gamma_m - \varepsilon_z)$ then the values of μ, ν exactly correspond to the misalignment angles according to (14). Observe that his remains true when the main angle is periodic instead of sinusoidal. As the angle γ in (8) is a good estimate for the main angle, one could use the inverse matrix based on this estimate rather than the average, but this gives less good results because the influence of the coupled components in (14) is accentuated in this case.

The calculation of the variables μ, ν is performed in two steps. From the product of the two matrices in (14)

$$R_y(\nu)R_x(\mu) = \begin{bmatrix} \cos(\nu) & \sin(\nu)\sin(\mu) & \sin(\nu)\cos(\mu) \\ 0 & \cos(\mu) & -\sin(\mu) \\ -\sin(\nu) & \cos(\nu)\sin(\mu) & \cos(\nu)\cos(\mu) \end{bmatrix}. \qquad (15)$$

one can calculate the angle μ with the elements (2,3) and (2,2).

$$\mu = -\mathrm{atan}\left(\frac{\left(R_z^{-1}(\gamma_m)R^0\right)_{2,3}}{\left(R_z^{-1}(\gamma_m)R^0\right)_{2,2}}\right). \qquad (16)$$

Next the mean value μ_m is used to calculate a modified matrix which is identified with a rotation matrix around the Y-axis

$$\left(R_z^{-1}(\gamma_m)R^0\right)R_x^{-1}(\mu_m) = R_y(\nu). \qquad (17)$$

This expression allows one to calculate the variable ν with

$$\nu = \mathrm{atan}\left(\frac{\left(R_z^{-1}(\gamma_m)R^0 R_x^{-1}(\mu_m)\right)_{1,3}}{\left(R_z^{-1}(\gamma_m)R^0 R_x^{-1}(\mu_m)\right)_{1,1}}\right). \qquad (18)$$

The mean value ν_m is obtained by averaging over one period of the signal. The mean values of the angles μ, ν around the X and Y axis, being estimates for the misalignment angles $\varepsilon_x, \varepsilon_y$, one can reorient the moving frame with respect to the joint frame using these angles.

B. The XYZ decomposition after correction

The global matrix R^0, based on the decomposition sequence YX chosen in (12) and (14), can now be corrected for the misalignment by multiplying with the inverse matrices

$$R^c = R^0 R_x^{-1}(\mu_m)R_y^{-1}(\nu_m). \qquad (19)$$

The XYZ decomposition of this matrix gives the angles α, β along the X and Y axis

$$\alpha = -\mathrm{atan}\left(\frac{R^c_{1,2}}{R^c_{2,2}}\right). \qquad (20)$$

$$\beta = \mathrm{asin}\left(R^c_{0,2}\right). \qquad (21)$$

These angles are compared with the coupled components in Fig. 2 and Fig.3.

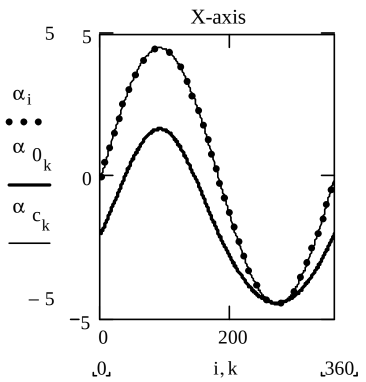

Fig.2 Cross-talk and coupled components along X

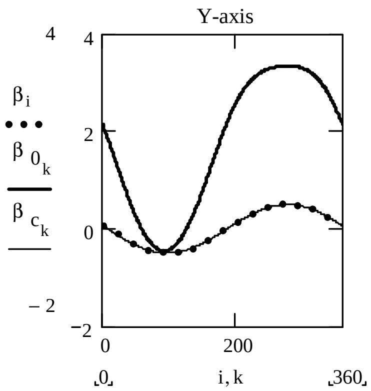

Fig.3 Cross-talk and coupled components along Y

The results (dotted lines) obtained with (20) and (21) practically coincide with the original coupled components in (3). The full lines shows the effect of the cross-talk in the components α_0, β_0 obtained from the global matrix R^0 with (7). This result shows that the misalignment of the moving frame with respect to the joint has been corrected. Misalignment of the joint with respect to the absolute reference frame can be corrected in the same way. In this case the misalignment matrix is multiplied with the joint matrix at the left, and the order of the decompositions is inverted. In practical situations one does however not know if the cross-talk is due to a misalignment of the joint with respect to the absolute frame or due to a misalignment of the moving frame with respect to the joint. In general one can expect both misalignments to contribute to the cross-talk. The correction for the misalignment is much more difficult in this case because the number of unknown doubles.

C. Main rotations around different axes

If the main rotation does take place around an axis different from the Z-axis, then one can apply the same procedure, but the expressions are different because the order of the decompositions is different. To apply the previous method one can also transform the initial rotation R_0 around the X or Y axis into a rotation around the Z-axis with the following permutation transform

$$R_z = P R_0 P^T \cdot \tag{22}$$

To transform from X to Z one has to use

$$P_{x \to z} = R_y(-90°) \cdot \tag{23}$$

and to transform from Y to Z

$$P_{y \to z} = R_x(90°) \cdot \tag{24}$$

III. TWO-SIDED TRANSFORM

The misalignment of both the joint with respect to the absolute frame and the moving frame with respect to the joint can be described by the following expression

$$R^0 = R(\xi) R_x(\alpha_c) R_y(\beta_c) R(\delta) R(\varepsilon) \cdot \tag{25}$$

In this case the method based on a multiplication with a matrix based on the mean value of the main angle as in (11) fails, although the main angle with (8) is well predicted from (25). Multiplying with the inverse matrix at the left gives bad estimates of the angles ε, and multiplication at the right gives bad estimates for the angles ξ. This is not unexpected, as to find the matrices $R(\varepsilon)$ and $R(\xi)$ one has to determine 6 unknowns. This problem can be solved by using a least-squares optimisation. If the hypothesis that the cross-talk effect always leads to an increase of the components along the X and Y axis is assumed true, then one can minimise these components to reduce the cross-talk. To avoid the convergence towards a solution which eliminates the components completely it was necessary to search a solution for which the main angle does not deviate too much from its first estimate with (8). The components along the X,Y and Z axes for an XYZ decomposition, given by (20),(21) and (8) are used.

To correct the misalignments, the original matrix in (25) is multiplied with a correction matrix at the left and at the right

$$R^c = R^{-1}(\sigma) R^0 R^{-1}(\tau) \tag{26}$$

When the misalignment matrices are exactly compensated by these correction matrices, then the following expression is minimised

$$\int_0^T \operatorname{atan}\left(\frac{R^c_{1,2}}{R^c_{2,2}}\right)^2 + \operatorname{asin}\left(R^c_{0,2}\right)^2 + \left(\gamma_0 + \phi + \operatorname{atan}\left(\frac{R^c_{0,1}}{R^c_{0,0}}\right)\right)^2 dt \tag{27}$$

Both correction matrices are written with the Rodrigues expression

$$\begin{aligned} R(\sigma) &= I + \sin(\sigma)\bar{u}^* + (1-\cos(\sigma))\bar{u}^*\bar{u}^* \\ R(\tau) &= I + \sin(\tau)\bar{v}^* + (1-\cos(\tau))\bar{v}^*\bar{v}^* \end{aligned} \tag{28}$$

Both adjoint matrices contain the unknown unit vectors $\bar{u}, \bar{v}$, giving the direction of the axis along which the correction angles σ, τ are taken. These contain three components of a unit vector subjected to the condition

$$|\overline{u}| = |\overline{v}| = 1 \qquad (29)$$

The parameter ϕ in (27) is introduced because the cross-talk effect can cause a difference between the initial guess γ_0 and the real angle δ . In total 7 parameters are found by optimisation. The initial values for all components are set to zero. The misalignment matrices used for the calculation are

$$\begin{aligned} R(\varepsilon) &= R_x(12°)R_y(-3°) \\ R(\xi) &= R_z(5°)R_y(7°)R_x(-5°) \end{aligned} \qquad (30)$$

The main angle δ is a sine-function with an amplitude of 20° and it has a mean value of 30° . The coupled components were phase-shifted and have amplitudes of 3° for α_c and 7° for β_c. Fig. 4 shows the main angle in comparison with the original values

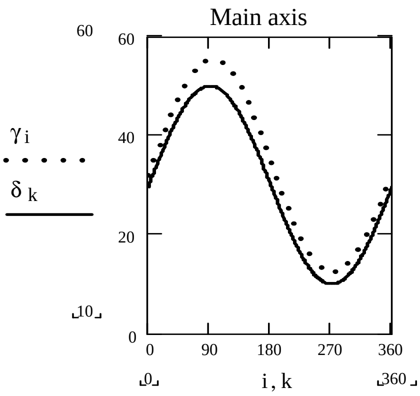

Fig.4 Main angle

The X and Y-components calculated from the original matrix R^0, and those calculated from the optimised matrix

$$R_{opt} = R^{-1}(\sigma_{opt})R^0R^{-1}(\tau_{opt}) \qquad (31)$$

(dotted line) are given in Fig.5 and Fig.6 in comparison with the ideal solution (full line). The cross-talk has largely been compensated.

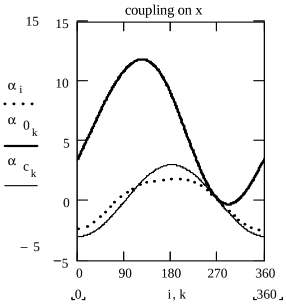

Fig.5 Cross-talk and coupled components along X

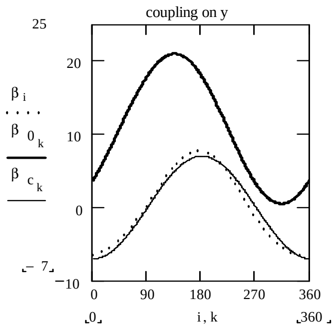

Fig.6 Cross-talk and coupled components along Y

This method also can be applied to the more simple case which was analysed in §II.

IV. Conclusion

A misalignment of a joint with respect to a fixed reference frame or the misalignment of a moving frame with respect to a joint can be compensated when the angles are periodic signals. In the case where both the moving frame and the joint are misaligned, a least-squares optimisation technique can be used to minimise the cross-talk effect.

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