Marker-Based Motion Reconstruction of Constrained Rigid-Segment Systems

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***Abstract*—This paper describes how a constrained nonlinear least-squares optimization approach can be used to recover the configuration of the segments in an arbitrary mechanical system from motion capture data. By appending the difference between markers in the model and the measured marker trajectories to the set of kinematic constraint equations, a set of over-determinate nonlinear equations will be obtained. This set of equations is solved by means of optimization. We also propose an Unscented Kalman Filter approach to the problem of configuration recovery and show how the velocity equations for a mechanical system subject to holonomic constraints can be re-written to a nonlinear state-space model. The formalism allows us to encode knowledge about f.x. smoothness, or any other knowledge the user might have about the system, into the filter.**

Keywords-component; Mechanical Systems, Motion Capture, Constrained Optimization, Unscented Kalman Filter.

I. Introduction

Marker-based motion capture is one of the most frequent tools used for measuring complicated motions to be used as input for musculoskeletal computer models. Such models are typically multi-body dynamics models, i.e. models in which the human body is idealized as a set of rigid segments connected by kinematical constraints, leaving a number of degrees of freedom, which must be prescribed e.g. by the motion capture data.

The motion reconstruction from motion capture data is inherently connected with a number of difficulties. As with all measurements, the data contains noise, skin artifacts and uncertainties on model parameters lead to discrepancies between model and measurement. Finally, more marker coordinates than degrees of freedom are needed to provide enough data to account for occasionally invisible markers and to provide reasonable accuracy of the independent set of degrees of freedom. Moreover, the model may be idealized to neglect certain motion (less significant) patterns. In the end, the measured marker positions provide an over-determinate (redundant) set of constraints for the model, which cause mathematical difficulties but also offer an opportunity to improve the accuracy of the experiment and the applicability of the results.

Various researchers have attacked this problem from different application points-of-view such as virtual reality [1], computer-based character animation [2], and biomechanical models [3]. In [1], a linearized version of a Kalman filter for tracking an articulated object in real-time is proposed. A slightly different version of the linear Kalman filter was also used in [4]. In [5], a solidification procedure to reduce the effects of skin-artifacts is developed, and other approaches, such as [6], have also tried to incorporate the determination of the skeletal parameters in the estimation process.

One of the drawbacks of the linearized Kalman filters is that the estimated result may not be particularly accurate – The Unscented Kalman Filter (UKF) has been shown to provide a better solution [7].

II. Basic Mechanics And Terminology

Suppose a mechanical system is composed of a set of N rigid segments, denoted $\{O^1,\dots,O^N\}$. The segments are moving in a workspace W, where $W \subseteq \Re^2$ in the 2D case and $W \subseteq \Re^3$ in 3D. A coordinate frame, $\{A\}$, is attached to the workspace. This coordinate frame will be referred to as the *global reference frame*. Reference frames are also attached to the segments, with $\{B_i\}$ being the reference frame for segment i. The *configuration space* for all segments in frame $\{A\}$ is hereby given by:

$$Q = Q^1 \times Q^2 \times \cdots \times Q^N \tag{1}$$

where each subspace, Q^i, represents the configuration of the ith segment. Define the dimension of Q as n. Let ${}^{B_i}s$ denote the coordinates of a point, s, in a local frame $\{B_i\}$, described in homogenous coordinates. In frame $\{A\}$, the coordinates of s will be described as:

$$ {}^{A}s = {}^{A}_{B_i}T(q)\,{}^{B_i}s \tag{2} $$

where ${}^{A}_{B_i}T(q)$ describes the coordinate transformation from frame $\{B_i\}$ to the global coordinate frame, $\{A\}$, in homogeneous coordinates, which includes both the translational and rotational displacements.

A. *Kinematic Constraints and Kinematic Drivers*

The motion of the segments may be subject to *kinematic constraints*. We shall assume that the constraints are *holonomic*, i.e. integratable, and can be written on the following form [8]:

$$ \Phi \equiv \Phi(q) = 0 \tag{3} $$

where $q \in Q$. Constraints written on this form reduce the number of degrees of freedom (DOF) of the system. Suppose there are $m < n$ independent kinematic constraint equations. The DOF of this system is then $k = n - m$.

If a set of k independent driver equations is additionally specified, a set of n equations in n unknowns is available. The driver equations have the following form:

$$ \Phi^{(d)} \equiv \Phi^{(d)}(q,t) = 0 \tag{4} $$

This set of nonlinear equations, (3) and (4), can be solved by a numerical approach, such as the Newton-Raphson method.

B. *Velocity Analysis*

By taking the derivative of (3) and (4) with respect to time, the velocity equations will be obtained:

$$ \Phi_q \dot{q} = 0 \tag{5} $$

$$ \Phi_q^{(d)} \dot{q} + \Phi_t^{(d)} = 0 \tag{6} $$

where Φ_q denotes the partial derivative with respect to q, also known as the Jacobian matrix, and $\Phi_t^{(d)}$ the partial derivative with respect to time.

III. MOTION CAPTURE

A *motion capture* system measures the position of markers attached to the segments in the global reference frame. The markers are partitioned into N sets belonging to the same segments:

$$ M = \bigcup_{i=1}^{N} S_i \tag{7} $$

where S_i is the set of markers attached to segment i, which has coordinate frame $\{B_i\}$ attached to it. The measurement of marker j, at discrete time k, in set S_i of markers is denoted ${}^{A}z_j^{S_i}(k)$. Once again, we use the convenient notation of homogeneous coordinates. The number of markers in set S_i is denoted M_i. In the model, the point that a marker measures may be expressed in local coordinates. Let the point in the model that marker ${}^{A}z_j^{S_i}(k)$ is a measurement for be denoted by ${}^{B_i}s_j$.

If a coordinate transformation, ${}^{A}_{B_i}T(q)$, is introduced that changes from local coordinates to global coordinates, the difference between the measured marker position and the point in the model is given by:

$$ e_j^{S_i}(k) = {}^{A}z_j^{S_i}(k) - {}^{A}_{B_i}T(q)\,{}^{B_i}s_j \tag{8} $$

IV. SOLUTION BY APPENDED DRIVER EQUATIONS

A direct approach to recover the configuration of the mechanical system, at each time step, would be to consider the differences between the markers in the model and the measured markers as driver constraints that should be driven to zero, i.e. $e_j^{S_i}(k) = 0$ for all S_i, j and k. However, due to various unavoidable inaccuracies not all differences can be driven identically to zero, such as noise in the measurements, skin artifacts and imprecise parameters in the model.

Instead, we suggest formulating the set of over-determinant nonlinear algebraic equations, given by (3) and (8), as a constrained nonlinear least-square optimization problem on the following form:

$$ \begin{aligned} &\min\left(\sum_{i=1}^{M_1} e_i^{S_1}(k)^T e_i^{S_1}(k) + \cdots + \sum_{i=1}^{M_N} e_i^{S_N}(k)^T e_i^{S_N}(k) \right) \\ &s.t.\ \Phi(q) = 0 \end{aligned} \tag{9} $$

This is in general a nonlinear, non-convex optimization problem, which may have several local minima. Since the parameters represent physical quantities, it is possible to provide a good initial guess – and local methods can be used to find the solution. We will address an example in Section VI.

V. SOLUTION BY UNSCENTED KALMAN FILTERING

In [7], a new extension to the well-known Kalman filter [9] for nonlinear systems was introduced. In this section, we show how partitioning the vector q into a dependent set and an independent set will allow us to rewrite the velocity equation in (5) to a nonlinear dynamic system on the form:

$$ \dot{q} = f(q,u) \tag{10} $$

$$ y = h(q,u) \tag{11} $$

for which Kalman filter theory can be applied. q is the state vector, and u is a yet unknown input vector. In the case of a mechanical system, q represents the configuration of the system, and u a vector, containing an independent set of velocities. First, we will review the main features of the UKF. For a complete explanation, please consult [7], and the references there in.

A. *The Unscented Kalman Filter*

The linear Kalman filter was originally designed as a recursive weighted least-square solution for estimating the state vector of a linear time-invariant system subject to Gaussian noise [9]. However, since not all models of physical systems are linear, it was first proposed to linearize the system equations and apply the linear Kalman filter to the linearized system. This approach is called Extended Kalman Filtering (EKF) [9]. Kalman filters work according to a specific "predictor-corrector" structure, where the system equations are used to predict the future state and observation, and the measurements (e.g. observations) are used to correct the estimate.

The idea behind the UKF is that it is easier to approximate a Gaussian distribution than an arbitrary nonlinear function [7]. In the prediction state in the UKF, a set of points, called *sigma points*, are propagated through the system model, and the weighted average of the propagated points are used as the prediction.

The UKF is designed for estimating the state vector for a discrete-time dynamic system of the form:

$$\begin{aligned} x(k+1) &= f\left[x(k), u(k), v(k), k\right], \\ z(k) &= h\left[x(k), u(k), k\right] + w(k) \end{aligned} \tag{12}$$

Where $x(k)$ is the state vector at time step k, $u(k)$ is the input vector, $v(k)$ is a process noise vector, $z(k)$ is the observation vector, and $w(k)$ is a measurement noise vector[1]. f and h are general vector fields, describing the system dynamics and the relationship between the state vector and the observations, respectively.

Before UKF can be applied to a mechanical system on the form introduced in Section II.A, the system equations must be rewritten to the form of (12). If the configuration vector q is partitioned into a dependent and an independent set, for which a new variable u is introduced, (e.g. $\dot{q}_j = u_i$), (5) can be re-written to the following form:

$$\dot{q} = \begin{bmatrix} \Phi_q \\ 0 \quad I \end{bmatrix}^{-1} \begin{bmatrix} 0 \\ u \end{bmatrix} \equiv f(q,u) \tag{13}$$

In (13), it is assumed that the entries of q have been re-arranged such that the lower part contains the independent variables. The velocity vector, u, is considered as a new input to the system. u can be used to encode knowledge about the motion into the filter, by appending additional differential equations to (13). For instance, this could be an assumption that the velocity is constant over a sample period, e,g, $\dot{u} = 0$.

[1] The process noise vector and the measurement noise vector prescribe how noise enters the model; in the difference-equations and the observation equations, respectively. The observation equations specify how measurements relate to the system dynamics.

Additionally, the observation equation can also be derived. For the measurement of the j th marker in set S_i, the following observation equation is available:

$$^{A}y_j^{S_i}(k) = {}^{A}_{B_i}T(q)\,^{B_i}s_j \equiv h(q,u) \tag{14}$$

As seen in (13) and (14), the noise vectors have been neglected. These formulas simply describe the mechanical system. If knowledge about how the noise enters the model is incorporated, the Kalman filter will be able to filter these out of the measured marker trajectories.

VI. Results

In this section, we will show how the above theories work on a simple mechanical system. The theory is applicable for any other mechanical system, consisting of rigid segments subject to holonomic constraints, for instance a human body model in 3D.

The example we will use is the so-called Crank-Slider mechanism. This system is composed of three segments, connected with three revolute joints, and a translational joint. The mechanical system is shown in Figure 1. The markers are numbered 1 to 9 from left to right. The complete set of equations describing this system can be found in [8].

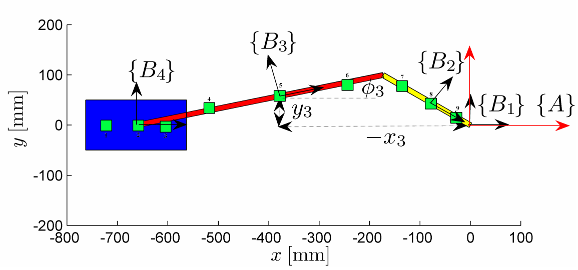

Figure 1. An illustration of the Crank-Slider mechanism. Only the definition of the configuration of the third local coordinate system is included. The others follow the same standard. The squares illustrate the marker positions.

For both the optimization approach and the UKF, ϕ_2 is used as the independent variable when the system is simulated, e.g. the marker trajectories are obtained by specifying a driver equation for ϕ_2. When the system has been simulated, the marker trajectories are created, and random noise is added. In this case, a zero-mean Gaussian noise is added with standard-deviation of 1 cm for all coordinates. A few of the markers are shown in Figure 2.

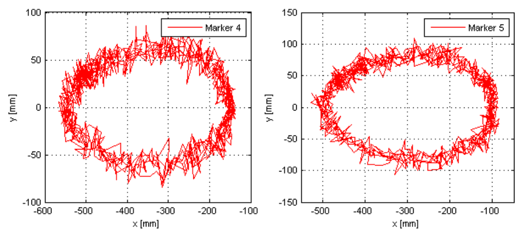

Figure 2. Markers 4 and 5 used to estimate the configuration of the crank-slider system. The remaining seven markers have the same amount of random noise added.

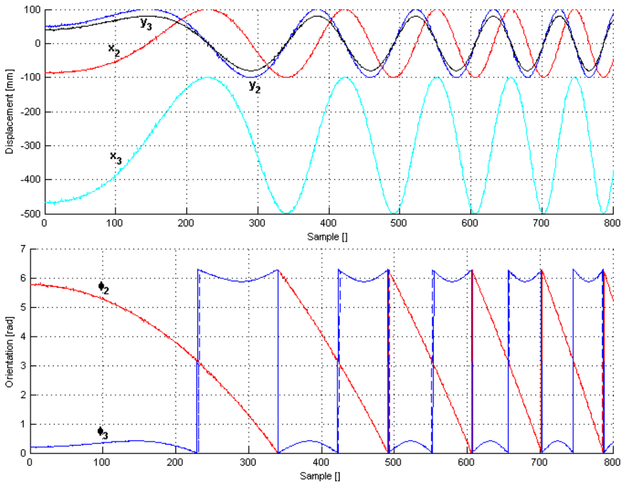

Figure 3. Simulated (solid lines) and estimated (dashed lines) result for x_2, y_2, ϕ_2, x_3, y_3, and ϕ_3, using the optimization approach.

A. The Constrained Optimization Approach

For the constrained optimization approach, the Sequential Quadratic Programming (SQP) approach is used to find the nearest local minimum of (9) from an initial guess. For each time step, the previous solution is used as the initial guess. Figure 3. shows the result of the estimation. As seen in the figure, the results are fairly accurate, although "high"-frequency noise is still present.

B. The Unscented Kalman Filter Approach

The same system is also solved using the UKF approach. A similar set of marker measurements are used. For this example, we define that $\dot{u} = K$ and $\dot{K} = 0$, and augment the vector q with these. Hereby, it is up to the filter to estimate both u and K in addition to q - with $\dot{\phi}_2 = u$.

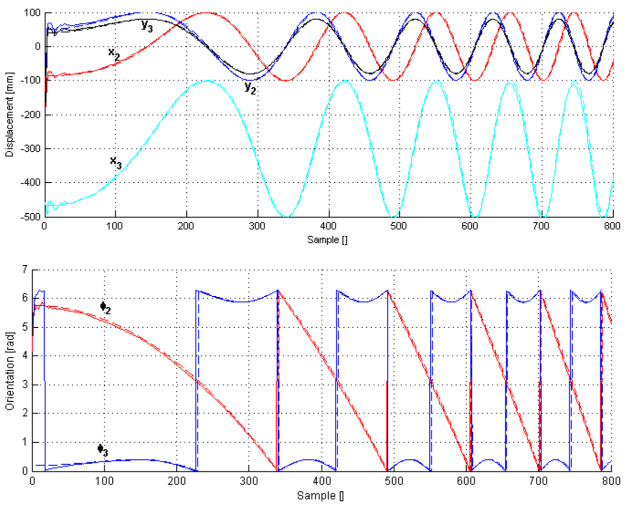

Figure 4. Estimate for x_2, y_2, ϕ_2, x_3, y_3, and ϕ_3, using UKF. The simulated is shown with solid lines and the estimates with dashed.

In Figure 4. , the estimation result is shown. As is seen in the figure, the recursive nature of UKF requires a settling time before the estimates are close to the correct state. For online applications, such as virtual reality, this initial propagation phase might be acceptable.

VII. Conclusion

As seen in the previous section, both methods manage to recover the configuration from marker trajectories. Both methods have some good features and some drawbacks.

The good feature of the optimization approach is that the estimation process does not introduce a settling period, where the estimate may be far from the correct state. On the other hand, the noise is not completely filtered out, which will cause problems if the result is differentiated for velocity or acceleration analysis. Additionally, the optimization solution is computationally expensive if hundreds of variables are involved.

In UKF, it is possible to incorporate accelerations, or similar information, and hereby get rid of the high frequency components. However, the drawback is that there will be a settling period, where the estimate is not particularly good. Computationally, UKF is the best choice since it only relies on matrix multiplications and similar operations. In that respect, it is suitable for real-time applications.

For an offline application, the optimization approach would be the best choice.

We believe that if the strong sides of both methods are combined, a much better estimate will be obtained. This is subject to ongoing research.

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