

Human upper limb kinematical model in a 3D workspace

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Abstract—This paper presents a kinematical model of the fingerless folding glove arm based on Denavit-Hartenberg convention from robotics. The model possesses all the motion freedoms and constrains of a real human limb, accommodating the modeling conditions imposed by the used convention. The active space described by the 3D hyper surfaces plotted in Matlab takes into account the natural limitation of the joints, according to the specialized literature, and was tested in several types of motion, covering all the basic flexion-extension, abduction-adduction, pronation-supination movements of the joints.

Keywords – upper limb, kinematical model; active workspace; 12 degrees of freedom

I. INTRODUCTION

For the complete kinematical study of the unrestrained human upper limb, based on anatomical considerations, a 12 degrees of freedom (DoF) open kinematical chain is considered. This model is named fingerless folding glove arm because the phalanxes all act simultaneously being able to gather the hand into a fist (fig 1).

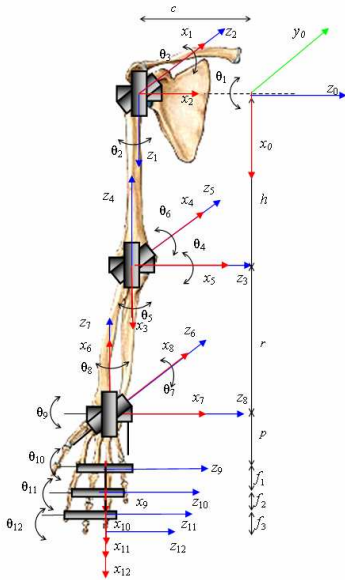


Figure 1. The complete kinematical model of the unrestrained upper limb

The $x_0O_0y_0z_0$ reference frame is considered with its origin in the point situated at the middle of the scapular belt. The other reference frames are placed in the joints having their origins displaced depending on the anatomical dimensions.

The joint succession was thus conceived as to assure a correct posture of the upper limb in orthostatic position. In this position the humerus-radius-palm-phalanx assembly is vertically placed at a distance, to the origin of the fixed reference frame, equal with the length of the clavicle and measured along the O_0z_0 axis (distance is positive for left limb and negative for the right one). All the components are situated in the $x_0O_0z_0$ frontal plane. The length of the humerus-radius-palm-phalanx assembly is oriented along the O_0x_0 axis.

II. COMPLETE KINEMATICAL MODEL

Table 1 of the joint variables is in accordance with figure 1 for the complete unrestrained upper limb model, the respective annotations refer the anatomical dimensions typical to the model:

- h length of humerus;
- r length of radius;
- p length carpal+metacarpal;
- f_i length of phalanx ($i=1, 2, 3$);
- c length of clavicle.

TABLE I.

Joint	θ_i	d_i	L_i	α_i
1	q_1	$-c$	0	90^0
2	q_2	0	0	90^0
3	q_3	0	h	90^0
4	q_4	0	0	-90^0
5	q_5	0	0	-90^0
6	q_6	0	r	0^0
7	q_7	0	0	90^0
8	q_8	0	0	90^0
9	q_9	0	p	0^0
10	q_{10}	0	f_1	0^0
11	q_{11}	0	f_2	0^0
12	q_{12}	0	f_3	0^0

Thus 12 transfer matrices (1) will be obtained, which multiplied will result the kinematical equations of the model.

$${}_{i-1}\mathbf{T}_i = \begin{bmatrix} \cos\theta_i & -\sin\theta_i \cos\alpha_i & \sin\theta_i \sin\alpha_i & l_i \cos\theta_i \\ \sin\theta_i & \cos\theta_i \cos\alpha_i & -\cos\theta_i \sin\alpha_i & l_i \sin\theta_i \\ 0 & \sin\alpha_i & \cos\alpha_i & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (1)$$

The general matrix \mathbf{G}_{12} , which represents the position and orientation of the reference frame attached to the phalanxes in regard to the fixed reference frame, is determined as product of the transfer matrices (2):

$$\mathbf{G}_{12} = {}^0\mathbf{T}_1 \cdot {}^1\mathbf{T}_2 \cdot {}^2\mathbf{T}_3 \cdot {}^3\mathbf{T}_4 \cdot {}^4\mathbf{T}_5 \cdot {}^5\mathbf{T}_6 \cdot {}^6\mathbf{T}_7 \cdot {}^7\mathbf{T}_8 \cdot {}^8\mathbf{T}_9 \cdot {}^9\mathbf{T}_{10} \cdot {}^{10}\mathbf{T}_{11} \cdot {}^{11}\mathbf{T}_{12} \quad (2)$$

Thus, the kinematical equations (3) obtained for the position of the finger's tip for upper limb are:

$$\begin{aligned} p_x &= f_3 \{ c_9_{10} c_{11} c_{12} [(((c_1 c_2 c_3 + s_1 s_3) c_4 + c_1 s_2 s_4) c_5 - (c_1 c_2 s_3 - s_1 c_3) s_5) c_7 \\ &+ ((c_1 c_2 c_3 + s_1 s_3) s_4 - c_1 s_2 c_4) s_7] - s_9_{10} c_{11} c_{12} [(((c_1 c_2 c_3 + s_1 s_3) s_4 - c_1 s_2 c_4) c_7 \\ &- ((c_1 c_2 c_3 + s_1 s_3) c_4 + c_1 s_2 s_4) c_5 - (c_1 c_2 s_3 - s_1 c_3) s_5) s_7] \} \\ &+ f_2 \{ c_9_{10} c_{11} [(((c_1 c_2 c_3 + s_1 s_3) c_4 + c_1 s_2 s_4) c_5 - (c_1 c_2 s_3 - s_1 c_3) s_5) c_7 \\ &+ ((c_1 c_2 c_3 + s_1 s_3) s_4 - c_1 s_2 c_4) s_7] - s_9_{10} c_{11} [((c_1 c_2 c_3 + s_1 s_3) s_4 - c_1 s_2 c_4) c_7 \\ &- ((c_1 c_2 c_3 + s_1 s_3) c_4 + c_1 s_2 s_4) c_5 - (c_1 c_2 s_3 - s_1 c_3) s_5) s_7] \} \\ &+ f_1 \{ c_9_{10} [(((c_1 c_2 c_3 + s_1 s_3) c_4 + c_1 s_2 s_4) c_5 - (c_1 c_2 s_3 - s_1 c_3) s_5) c_7 \\ &+ ((c_1 c_2 c_3 + s_1 s_3) s_4 - c_1 s_2 c_4) s_7] - s_9_{10} [((c_1 c_2 c_3 + s_1 s_3) s_4 - c_1 s_2 c_4) c_7 \\ &- ((c_1 c_2 c_3 + s_1 s_3) c_4 + c_1 s_2 s_4) c_5 - (c_1 c_2 s_3 - s_1 c_3) s_5) s_7] \} \\ &+ p \{ c_9 [(((c_1 c_2 c_3 + s_1 s_3) c_4 + c_1 s_2 s_4) c_5 - (c_1 c_2 s_3 - s_1 c_3) s_5) c_7 \\ &+ ((c_1 c_2 c_3 + s_1 s_3) s_4 - c_1 s_2 c_4) s_7] - s_9 [((c_1 c_2 c_3 + s_1 s_3) s_4 - c_1 s_2 c_4) c_7 \\ &- ((c_1 c_2 c_3 + s_1 s_3) c_4 + c_1 s_2 s_4) c_5 - (c_1 c_2 s_3 - s_1 c_3) s_5) s_7] \} + r \{ ((c_1 c_2 c_3 + s_1 s_3) c_4 \\ &+ c_1 s_2 s_4) c_5 - (c_1 c_2 s_3 - s_1 c_3) s_5 \} + h \{ (c_1 c_2 c_3 + s_1 s_3) \} \\ p_y &= f_3 \{ c_9_{10} c_{11} c_{12} [(((s_1 c_2 c_3 - c_1 s_3) c_4 + s_1 s_2 s_4) c_5 - (s_1 c_2 s_3 + c_1 c_3) s_5) c_7 \\ &+ ((s_1 c_2 c_3 - c_1 s_3) s_4 - s_1 s_2 c_4) s_7] + s_9_{10} c_{11} c_{12} [(((s_1 c_2 c_3 + c_1 s_3) s_4 + s_1 s_2 c_4) c_7 \\ &+ (((s_1 c_2 c_3 - c_1 s_3) c_4 + s_1 s_2 s_4) c_5 - (s_1 c_2 s_3 + c_1 c_3) s_5) s_7] \} \\ &+ f_2 \{ c_9_{10} c_{11} [(((s_1 c_2 c_3 - c_1 s_3) c_4 + s_1 s_2 s_4) c_5 - (s_1 c_2 s_3 + c_1 c_3) s_5) c_7 \\ &+ ((s_1 c_2 c_3 - c_1 s_3) s_4 - s_1 s_2 c_4) s_7] + s_9_{10} c_{11} [(((s_1 c_2 c_3 + c_1 s_3) s_4 + s_1 s_2 c_4) c_7 \\ &+ (((s_1 c_2 c_3 - c_1 s_3) c_4 + s_1 s_2 s_4) c_5 - (s_1 c_2 s_3 + c_1 c_3) s_5) s_7] \} \\ &+ f_1 \{ c_9_{10} [(((s_1 c_2 c_3 - c_1 s_3) c_4 + s_1 s_2 s_4) c_5 - (s_1 c_2 s_3 + c_1 c_3) s_5) c_7 \\ &+ ((s_1 c_2 c_3 - c_1 s_3) s_4 - s_1 s_2 c_4) s_7] + s_9_{10} [(((s_1 c_2 c_3 + c_1 s_3) s_4 + s_1 s_2 c_4) c_7 \\ &+ (((s_1 c_2 c_3 - c_1 s_3) c_4 + s_1 s_2 s_4) c_5 - (s_1 c_2 s_3 + c_1 c_3) s_5) s_7] \} \\ &+ p \{ c_9 [(((s_1 c_2 c_3 - c_1 s_3) c_4 + s_1 s_2 s_4) c_5 - (s_1 c_2 s_3 + c_1 c_3) s_5) c_7 + ((s_1 c_2 c_3 - c_1 s_3) s_4 \\ &- s_1 s_2 c_4) s_7] + s_9 [(((s_1 c_2 c_3 + c_1 s_3) s_4 + s_1 s_2 c_4) c_7 + (((s_1 c_2 c_3 - c_1 s_3) c_4 + s_1 s_2 s_4) c_5 \\ &- (s_1 c_2 s_3 + c_1 c_3) s_5) s_7] \} + r \{ ((s_1 c_2 c_3 - c_1 s_3) c_4 + s_1 s_2 s_4) c_5 - (s_1 c_2 s_3 + c_1 c_3) s_5 \} \\ &+ h \{ (s_1 c_2 c_3 - c_1 s_3) \} \\ p_z &= f_3 \{ c_9_{10} c_{11} c_{12} [(((s_2 c_3 c_4 - c_2 s_4) c_5 - s_2 s_3 s_5) c_7 + (s_2 c_3 s_4 + c_2 c_4) s_7] - \\ &- s_9_{10} c_{11} c_{12} [(((s_2 c_3 s_4 + c_2 c_4) c_7 - ((s_2 c_3 c_4 - c_2 s_4) c_5 - s_2 s_3 s_5) s_7) \} + \\ &+ f_2 \{ c_9_{10} c_{11} [(((s_2 c_3 c_4 - c_2 s_4) c_5 - s_2 s_3 s_5) c_7 + (s_2 c_3 s_4 + c_2 c_4) s_7] - \\ &- s_9_{10} c_{11} [(((s_2 c_3 s_4 + c_2 c_4) c_7 - ((s_2 c_3 c_4 - c_2 s_4) c_5 - s_2 s_3 s_5) s_7) \} + \\ &+ f_1 \{ c_9_{10} [(((s_2 c_3 c_4 - c_2 s_4) c_5 - s_2 s_3 s_5) c_7 + (s_2 c_3 s_4 + c_2 c_4) s_7] - \\ &- s_9_{10} [(((s_2 c_3 s_4 + c_2 c_4) c_7 - ((s_2 c_3 c_4 - c_2 s_4) c_5 - s_2 s_3 s_5) s_7) \} + \\ &+ p \{ c_9 [(((s_2 c_3 c_4 - c_2 s_4) c_5 - s_2 s_3 s_5) c_7 + (s_2 c_3 s_4 + c_2 c_4) s_7] - \\ &- s_9 [(((s_2 c_3 s_4 + c_2 c_4) c_7 - ((s_2 c_3 c_4 - c_2 s_4) c_5 - s_2 s_3 s_5) s_7) \} + \\ &+ r \{ ((s_2 c_3 c_4 - c_2 s_4) c_5 - s_2 s_3 s_5) \} + h \{ s_2 c_3 - c \} \end{aligned} \quad (3)$$

The orthostatic position of the upper limb was represented considering all the joint variables being null (fig 2).

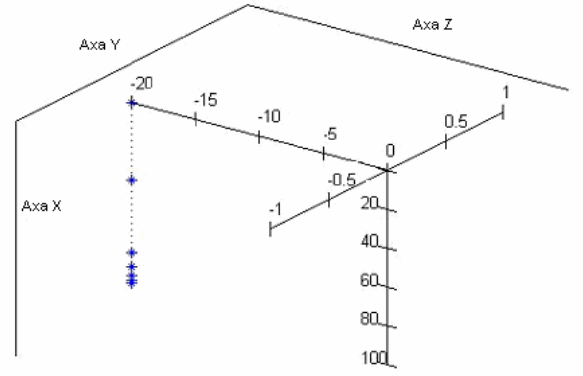


Figure 2. The human upper limb in orthostatic position

Accepting some lengths: $h = 30\text{cm}$, $r = 25\text{cm}$, $p = 10\text{cm}$, $f_1 = 4.5\text{cm}$, $f_2 = 2.5\text{cm}$, $f_3 = 1.5\text{cm}$ for the bone segments, it is observed that in this position, the upper limb is aligned along the line of equation $y = 0$, $z = -20\text{cm}$ the extremity of the fingers being placed at coordinate $x = h + r + p + f_1 + f_2 + f_3 = 73.5\text{cm}$. The ranges that define the joint variables have been stated so that movement of the upper limb's model may be as natural as possible: $-110^\circ \leq q_1 \leq 30^\circ$, $-45^\circ \leq q_2 \leq 45^\circ$, $0^\circ \leq q_3 \leq 135^\circ$, $-140^\circ \leq q_4 \leq 0^\circ$, $-45^\circ \leq q_5 \leq 90^\circ$, $-4^\circ \leq q_6 \leq 4^\circ$, $-55^\circ \leq q_7 \leq 30^\circ$, $-4^\circ \leq q_8 \leq 4^\circ$, $-90^\circ \leq q_9 \leq 80^\circ$, $-45^\circ \leq q_{10} \leq 90^\circ$, $0^\circ \leq q_{11} \leq 90^\circ$ and $0^\circ \leq q_{12} \leq 90^\circ$.

III. ACTIVE WORKSPACE

The representation of the upper limb's active workspace was made both in simplified models and in a model in which all the kinematical joints execute simultaneous movements in angular ranges allowed by the joint configurations. Due to the complete model's complexity, a model of the active workspace was made in Matlab with only 3 division per angular interval (figure 3), further division of the interval leading to an unjustified increase in the number of operations without a significant increase in the results. Even so, for this model the number of points computed and represented is equal to 59048.

It is observed that the extreme limits of the hyper surface that delimits the active space do not exceed on the three axis the value $\sqrt{(h+r+p+f_1+f_2+f_3)^2 + c^2} = 76.17\text{cm}$, extension along the $O_0 z_0$ axis being theoretically comprised between the positive value of $\sqrt{(h+r+p+f_1+f_2+f_3)^2 + c^2} - c = 56.17\text{cm}$ and maximum negative value of $-(\sqrt{(h+r+p+f_1+f_2+f_3)^2 + c^2} + c) = -96.17\text{cm}$, obtained due to the shoulder's joint placement at $z = -20\text{cm}$.

Practically, the negative maximum can be attained easily when the limb is extended in horizontal position outwards in the frontal plane, regardless of the imposed constraints to the joint variables, but on the other hand, the distal end cannot exceed the value determined by the sum of the segments

$c + \frac{h}{3} = 30cm$ (verifiable through horizontal extension of the other upper limb which is placed fully in the positive direction of the O_0z_0 axis).

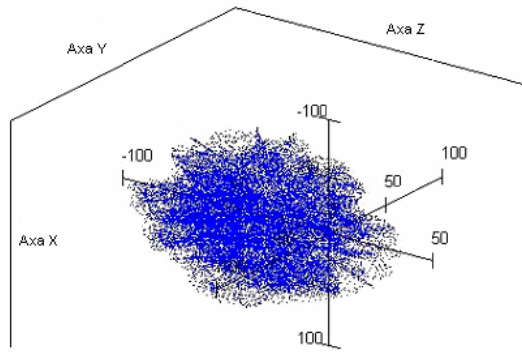


Figure 3. The active space of the upper limb when the angular domains are divided into three intervals

Same as for the lower limb, the hyper surfaces of the upper limb can be represented when movement is made only in some kinematical joints. For example, if in turns the movement is stopped in the joints that model the scapulohumeral joint, but in the same time movement is allowed in all the other joints, different hyper surfaces are obtained illustrating:

- The active space when movement is made only in the first joint, that is, only flexion-extension is allowed (figure 4);
- The active space when just the second joint moves, that is, only internal-external rotation is allowed (figure 7);
- The active space when just the third joint moves, that is, only abduction-adduction is allowed (figure 9).

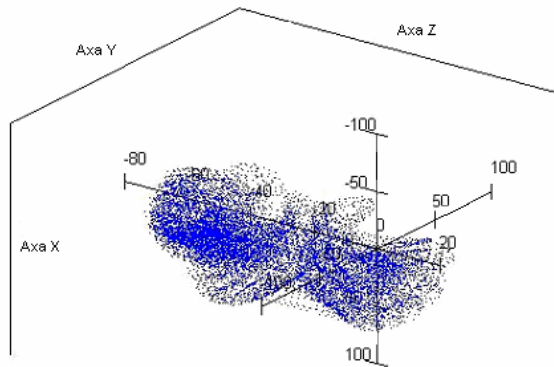


Figure 4. The active space of the upper limb during flexion-extension movement in the scapulohumeral joint.

In all the cases, the active joint's domain from the scapulohumeral joint was divided into 10 intervals, all the other joints being divided into 2 intervals. At each of these movements the hyper surface's projection in perpendicular plane on its rotation axis was also represented, each angular division being initially divided into 3 intervals.

Thus, during flexion-extension movement made by the first joint, projection in the fix plane $x_0O_0y_0$ shows that the maximum value of the vertical displacement is not recorded along the O_0x_0 axis in its positive direction, but somewhere along the vertical line of equation $y=30cm$. Moreover the maximum value of the x coordinate for the finger's tip is not 73.5 cm as resulted by the sum of the anatomic segments on vertical, but approximately 63 cm (figure 5). This is due only to the lack of precision of this representation. If the precision is increased by dividing the domains for joints 1, 4, 5, 6 into 10 intervals and the rest into 2 intervals, the representation will have 24057 points and the precision will increase accordingly (figure 6).

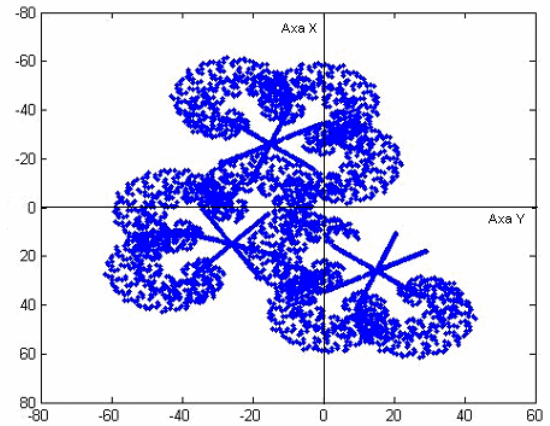


Figure 5. Projection in sagittal plane of the active space for the upper limb during flexion-extension movement in the scapulohumeral joint (2673 points)

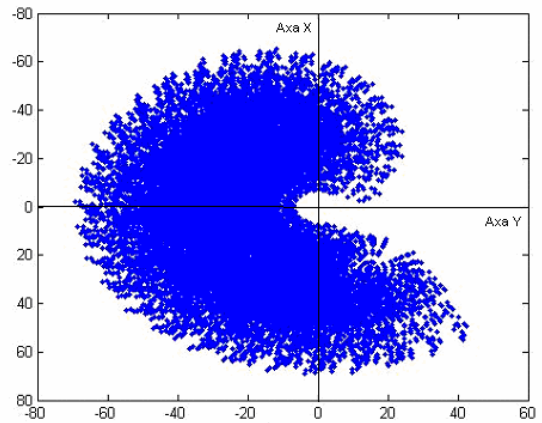


Figure 6. Projection in sagittal plane of the active space for the upper limb during flexion-extension movement in the scapulohumeral joint (24057 points)

It is observed that, in this representation, the maximum extension of the upper limb is produced when it passes through vertical position, recording at the finger's tip a value of the coordinate $x_{max} = 73.5cm$. The projection of the displacement along the O_0y_0 axis is naturally larger in the negative direction of the axis than in the positive one. It is

observed that the finger's tip is placed in the sagittal plane at a maximum horizontal distance of $y_{\max} = x_{\max} = 73.5\text{cm}$ to the shoulder joint. If in the scapulohumeral joint only the second joint is moving and the other joints (from 4 to 12) are moving normally, the active hyper surface is presented in figure 7, and its projection in the $y_0O_0z_0$ plane has the shape from figure 8. The representation conditions are those corresponding to figure 4 and 6. It is observed that the projection of the scapulohumeral joint in transversal plane has the coordinates $y=0$ $z=-20\text{cm}$ and in the domain defined by $y<0$ the density of positions of the distal end is much bigger than for the domain $y>0$, which is normal if taken into account the natural gesture of the upper limb. The covered area through motion is practically symmetric to a line of equation $z=-20$ in the domain with $y<0$. The movement in the transversal plane presents a pronounced asymmetry to the O_0z_0 axis, explained by the fact that the finger's tip can occupy far fewer positions in internal rotation than in external rotation.

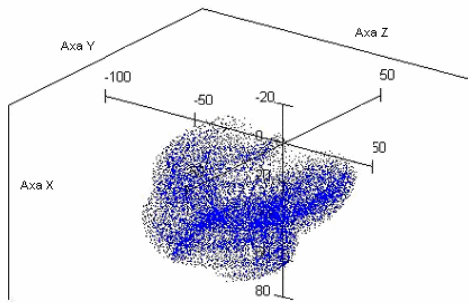


Figure 7. The active space of the upper limb during internal-external rotation movement in the scapulohumeral joint

The hyper surface that delimits the active space during the abduction-adduction movement in the scapulohumeral joint is represented in figure 8.

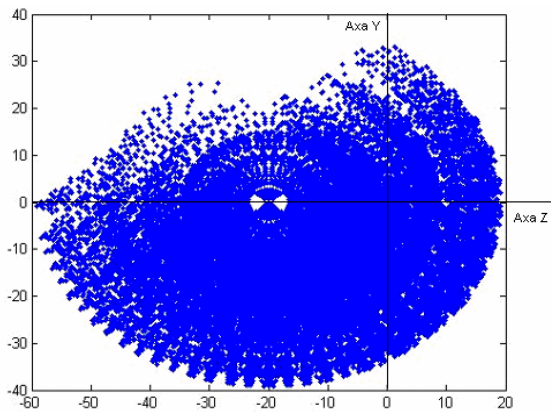


Figure 8. Projection in transversal plane of the active space for the upper limb during internal-external rotation movement in the scapulohumeral joint

The projection of this hyper surface in $x_0O_0z_0$ plane was represented by dividing the domain of joints 3, 4, 5 and 6 in 10

intervals, all the other joints having the domains divided into 2 intervals. (figure 10).

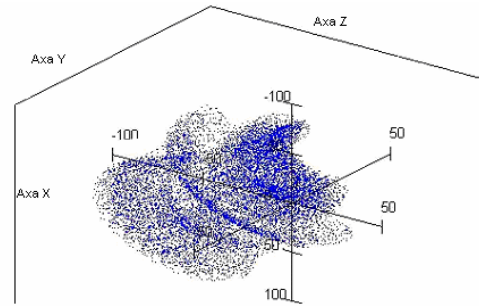


Figure 9. The active space of the upper limb during abduction-adduction movement in the scapulohumeral joint

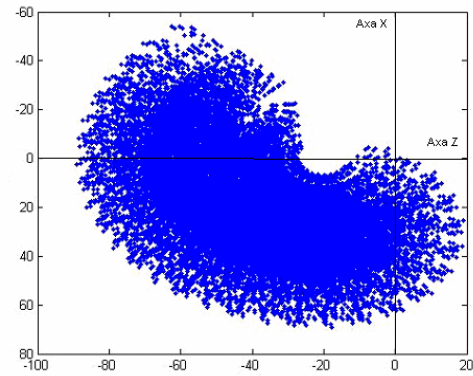


Figure 10. Projection in the frontal plane of the active space for the upper limb during internal-external rotation movement in the scapulohumeral joint

It is observed that the projection of the scapulohumeral joint in frontal plane has coordinates $x=0$ $z=-20\text{cm}$ and the maximum extent along O_0z_0 axis is placed at a distance of 93.5cm in negative direction, which corresponds to the position to the horizontally extended upper limb in abduction. In this case, the finger's tip is placed at a maximum distance of $93.5-20=73.5\text{cm}$ from the joint, corresponding to the sum of the anatomical segments $h+r+p+f_1+f_2+f_3$. In the position characterized by coordinate $z=-20\text{cm}$ (in line with the scapulohumeral joint) the maximum value of the vertical movement is recorded, 73.5cm in the positive direction of the O_0x_0 axis as in the orthostatic position.

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