Robust estimation of screw axis from 3D pose using dual quaternion algebra

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***Abstract*—The purpose of this study is to investigate the accuracy of a new algorithm based on dual quaternion algebra for the estimation of the finite screw axis .**

Keywords-component; finite screw axis, dual quaternion algebra, vector algebra, 3-D pose, knee joint kinematics.

I. Introduction

Increasing efforts in properly quantifying 3-D joint kinematics for unambiguous comparison of normal and pathological movement such as knee-osteoarthritis [1-2], and knee ligament rupture [3-4], call for suitable and calculation procedures. The survey in [5] shows that the screw-home motion of the knee occurs in general on six displacement axes; i.e. the translation axes are not coincident with the rotation axes, and also that the three rotation axes are skewed and do not intersect. However, the measurement of the screw-home motion at the knee may be strongly influenced by errors in the location of the axis of rotation, and due only to kinematics crosstalk [6]. The general six degrees of freedom continuous motion between two rigid bodies is fully described by the so-called instantaneous screw axis (ISA) [7]. Hence, the instantaneous and relative motion of the body under study is fully described by the line *L* and the rate of rotations about and sliding along *L*, reason why this line is termed instantaneous screw axis. In many biomechanical motion studies, however, the accuracy of the ISA had limited its use since it requires the knowledge of the first derivative of the displacement signal. In general, a static approach is used which describes the discrete relative motion of the body by a finite rotation around and finite translation about a fixed axis called the finite screw axis (FSA) [8].

Woltring et al. [8] developed analytical expression of the standard deviation of the FSA parameters and simulated numerically an isotropic distribution case. He found that the error in the direction of the FSA was half time the rotation magnitude, whereas the error on the position of the FSA was ten times the error of the landmarks measurement system [8]. In an experimental simulation study of one degree rotation of four co-planar markers, the standard deviation of the orientation of the FSA vary by 6° whereas the standard deviation of the FSA's position vary between 60 to 300 times the standard deviation of the landmarks measurement system [9]. These uncertainties on the estimation of the orientation and the position necessitate generally an over smoothing procedure for the displacement data, which can strongly distort the original landmark trajectories. FSA accuracy has been assessed in a numerical simulation of knee joint during gait, and it was found that the minimum threshold value of 20° between successive 3D positions, is necessary to keep the orientation and the position errors of the FSA under 10° and 1 cm respectively [10]. Moreover, the authors [10] found in an experimental study of the functional axes of the elbow joint that the minimum required threshold to keep the FSA orientation and position errors respectively under 3° and 1 cm, is about 45°. Finally the authors [10] concluded that the FSA is not a good way to describe the variation of a joint displacement during a given movement. It should be noted however that Chèze et al. [10] used the Rodrigues' formula which is very sensitive to noisy data as mentioned originally in [11]. Although the vector algebra method originally devised in [12], had a poor performance [13], it is still in used nowadays [14]. The technique used in [14] fails if the vector displacement of any single maker between the initial and the final position, becomes parallel or nearly colinear to the FSA. A consistent method such a dual quaternion algebra that estimate a dual quaternion which represents simultaneously a rotation and translation of rigid body in 3D space have been shown to perform better [15] than the singular value decomposition method [16]. The purpose of this study is to compare the dual quaternion algebra method to the vector algebra during a passive in-vitro movement of knee cadaveric specimens.

II. Methods

The following section will briefly described the two methods as well as the description of the experimental set-up. The following notation is used for all eight algorithms:

- *e*: 3x1 column vector, defining the orientation of the finite screw axis.
- *s*: 3 x 1 column vector, coordinates of a point located on the FSA.
- *u*: a scalar defining the translation magnitude along the FSA.
- θ: a scalar defining the angle of rotation about the FSA.

- x_i, y_i : 3x1 column vector, coordinates of the point i=1,..,n of the moving rigid body in the first and second configuration.
- X,Y : 3xn distribution matrix,
  $X = [x_1, x_2, \cdots, x_n]; \quad Y = [y_1, y_2, \cdots, y_n]$
- $\bar{x} = \frac{1}{n}\sum x_i; \quad \bar{y} = \frac{1}{n}\sum y_i$: centroid of the points in the first and second configuration
- *Vect* operator transforms a 3x3 skew-symmetric matrix V to a 3D vector v by:
- $v = Vect(V) = \frac{1}{2}\begin{bmatrix} V_{32} - V_{23} \\ V_{13} - V_{31} \\ V_{21} - V_{12} \end{bmatrix}$
- *Tr* : trace of matrix
- *Sk* operator transforms a vecteur $v = [v_1 \quad v_2 \quad v_3]^T$ into a skew-symmetric matrix V such as: $V = Sk(v) = \begin{bmatrix} 0 & -v_3 & v_2 \\ v_3 & 0 & -v_1 \\ -v_2 & v_1 & 0 \end{bmatrix}$

.*Vector Algebra method (BG)*

The method is based on marker cluster geometry. First the matrix A is computed by:

$$A = (X - Y)^{-T}$$

A scale factor is also computed from the matrix element A_{ij} as:

$$scale = 1/sqrt((A_{11}+A_{12}+A_{13})^2 + (A_{21}+A_{22}+A_{23})^2 + (A_{31}+A_{32}+A_{33})^2)$$

The orientation vector e is determined from A and *scale*:

$$e = scale \times A.h$$

where h is a 3x1 unit tensor. The rotation angle θ is calculated from:

$$\cos(\theta) = (V_1^T V_2 - (V_1^T e)^2)/(V_1^T V_1 - (V_1^T e)^2)$$

With $: V_1 = x_1 - x_2$ and $V_2 = y_1 - y_2$

The rotation matrix is computed:

$$R = \begin{bmatrix} \cos(\theta)+v(\theta)e_x^2 & -\sin(\theta)e_z+v(\theta)e_y e_x & \sin(\theta)e_y+v(\theta)e_z e_x \\ \sin(\theta)e_z+v(\theta)e_y e_x & \cos(\theta)+v(\theta)e_y^2 & -\sin(\theta)e_x+v(\theta)e_y e_z \\ -\sin(\theta)e_y+v(\theta)e_x e_z & \sin(\theta)e_x+v(\theta)e_y e_z & \cos(\theta)+v(\theta)e_z^2 \end{bmatrix}$$

$$v(\theta) = 1 - \cos(\theta)$$

The orientation vector e of the helical axis and the rotation angle θ about it are extracted from the rotation matrix R as

$$e = \frac{Vect(R)}{\|Vect(R)\|}; \quad \cos(\theta) = \frac{(Tr(R)-1)}{2}; \quad \sin(\theta)e = vect(R)$$

The translation vector is obtained from:

$$t = \bar{y} - R\bar{x}$$

The position vector s of the helical axis is obtained by the projection of the midpoint p on the finite displacement vector d:

$$s = p + \{2\tan(\theta/2)\}^{-1}.(e \times d); \quad p = (\bar{x}+\bar{y})/2; \quad d = \bar{y} - \bar{x};$$

$$P1_i = [x_i/2;0] \quad P2_i = [y_i/2;0]$$

The magnitude u of the translation along the helical axis is equal to

$$u = e^T(\bar{y} - \bar{x}).$$

.*Dual quaternion algebra method (DQ)*

The rotation matrix is expressed by a quaternion whereas the translation vector is expressed by another quaternion (together they form the Plücker line). First, the 3D markers are transformed in the quaternion space:
Three matrices are constructed from $P1_i$ and $P2_i$:

$$C_1 = -2\sum_{i=1}^{3} Q(P2_i)^T W(P1_i)$$

$$C_2 = 3I_d$$

$$C_3 = 2\sum_{i=1}^{3} (W(P1_i)^T Q(P2_i))$$

For every quaternion $q = [q_s, q_v]^T$ where qs and q_v represent scalar and vector parts respectively. The following matrices W and Q are defined by:

$$W(q) = \begin{bmatrix} q_s I_d - sk(q_v) & q_v \\ -q_v^T & q_s \end{bmatrix} \qquad Q(q) = \begin{bmatrix} q_s I_d + sk(q_v) & q_v \\ -q_v^T & q_s \end{bmatrix}$$

The matrix A is then computed from C_i as:

$$A = \frac{1}{2}[C_3^T (C_2 + C_2^T)^{-1} C_3 - C_1 - C_1^T]$$

We compute the eigenvector r corresponding to the largest positive eigenvalue of matrix A. From r we can derive s as:

$$s = -(C_2 + C_2^T)^{-1} C_3 r$$

The rotation matrix and the translation vector are calculated from:

$$\begin{bmatrix} R & 0 \\ 0 & 1 \end{bmatrix} = W(r)^T Q(r) \qquad t = W(r)^T s$$

The orientation vector is obtained from the vector part of the quaternion r as

$$e = \frac{r}{\|r\|}$$

The rotation angle is computed from:

$$\cos(\theta) = r_s^2 - \|r\|^2 \qquad \sin(\theta) = 2 r_s \|r\|$$

*.**Experimental set-up***

Eight specimens of cadaver knee have been used in this study. The details information of this database can be found in [3]. Three radio-reflective metal triangles were fixed on the femur and the tibia of the specimen. The specimen was first CT-Scanned and then the triangles were digitized during a movement using a an electromagnetic six degrees of freedom instrumentation at 40Hz (Flock-Birds) in a slow passive flexion/extension movement induced by the operator's hand. Figure 1 shows a typical knee segment CT-scanned with the metallic reflective triangles. During the movement only the extension part was extracted and the FSA parameters were computed between each successive frame. The mean screw axis was then computed with a optimal formula as proposed in [8]. the dispersion of the three FSA parameters with respect to the MSA have been estimated.

*.**Statistical analysis***

The following dependant parameters DispOri, DisPos, MT which represent respectively the dispersion in orientation and position as well as the mean translation with respect to the MHA has been used in an ANOVA to detect a significant difference between DQ and BG method for estimating the FSA.

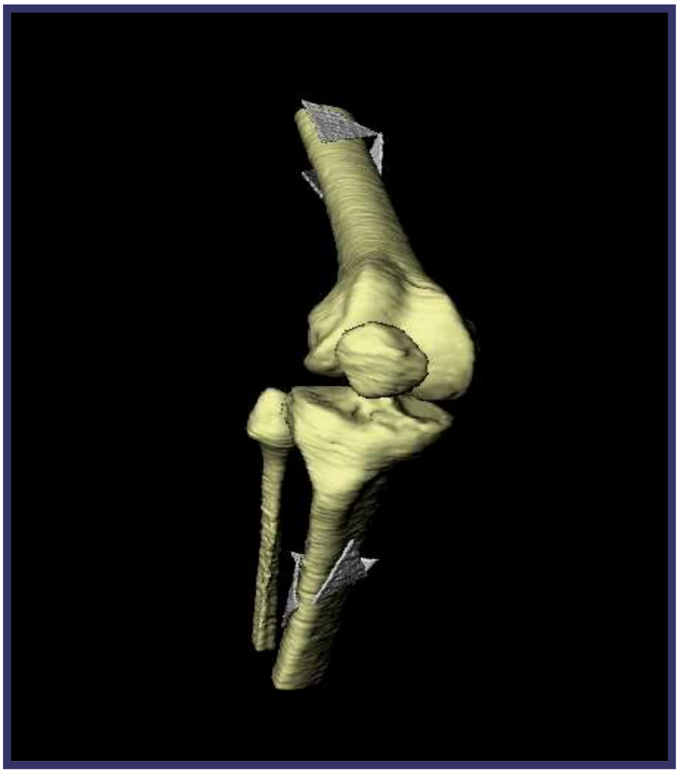

Figure 1: CT-scan of a specimen knee with metallic triangles

III. RESULTS

During the manipulation of the flexion-extension movement by the operator a force of approximately 70 N was applied to the hamstring muscle structure of the specimens. The dispersion in position of the FSA is presented in Table 1. From specimen 1 to 8 it ranged from 22.09 mm to 111.58 mm when using BG method. However, the dispersion in position ranged from 24.79 to 57.01 when using DQ algorithm. There was a significant difference between DQ and BG algorithms ($p < 0.05$). Table 2 shows results on the dispersion in orientation with respect the MHA. When using BG algorithm the dispersion in orientation ranged from 9.31 deg to 43.45 degrees, whereas it varies between 8.47 and 16.03 degrees. There was a significant difference between BG and DQ methods ($p < 0.05$). The dispersion in translation varies from 0.24 to 6.37 mm for BG algorithm, whereas it varies form -0.21 to 0.65 mm using DQ algorithm. There was a significant difference in the dispersion in translation between DQ and BG methods ($p < 0.05$).

Table 1: Dispersion in position with respect to the MHA. (Units are in mm)

Specimen	***BG***	***DQ***
1	111.58	28.24
2	49.75	25.77
3	87.24	57.01
4	22.09	24.79
5	59.05	42.84
6	44.77	40.94
7	184.70	32.17
8	121.72	35.00
Mean	85.11	35.85
Std	52.77	10.82

Table 2: Dispersion in orientation with respect to the MHA. (Units are in degrees)

Specimen	***BG***	***DQ***
1	20.37	8.47
2	15.48	7.69
3	11.38	10.47
4	9.31	9.29
5	14.53	14.51
6	16.17	16.03
7	40.29	13.40
8	43.45	11.10
Mean	21.37	11.37
Std	13.09	3.00

IV. DISCUSSION

The purpose of this study was to investigate the effect of using two different algorithms to estimate the FHA parameters. Our data show a high variability of the parameter related to the dispersion in the position for the BG algorithm, whereas as the DQ one exhibits more repeatability. Blankevoort et al. [17] reported on the intersection of the FSA axis with the medial and sagittal planes. Their value reached the maximal value of approximately 50 mm for their four specimens. This is an agreement with method DQ in our case. It is difficult to compare our orientation data to that in [17]. Also the translation was reported in [17], it was expressed as a pitch parameter i.e. the ratio between the rotational and translational quantity. The translational information reported in [18] varies from 0.3 to1.3 mm for fourteen normal subjects using stereo-radiographs of different position of the knee. The results of [18] are closer to the data obtained by our study in the case of DQ algorithm. Jonsson and Kärrholm [19] reported on the inclination of the FSA with respect to the frontal plane. Their value form 1.2 to 26.6 degrees, which is closer to the DQ method. It should be noted that in [19] data was reported on different intervals of knee flexion, whereas in ours data was reported with respect to the mean screw axis.

The advantages of using finite screw axis to describe the kinematics of the knee cadaver specimens is that is rotation and translation around the screw axis are independent from the coordinate system. However, the orientation and the position of the FSA depend on the coordinate system. In fact, it has been already shown that the choice of coordinate system influence the knee kinematics and can create a substantial cross-talk between the movement of flexion/extension and internal/external rotation. This has been confirmed recently in cadaver study in using different anatomical axes of rotation [20]. In our study we use the geometric center of the femoral epycondyle for the establishment of our reference system. However, in our knowledge, this is the first study that compares the use of different techniques to assess the FSA. The method proposed in Bisshipp [11] and Begg [12] are related first to perfect data i.e. without noise and in a situation where there are no singularities (i.e. marker displacement parallel to the FSA). The last condition is difficult to guarantee. The DQ algorithm is safe from any singularities and incorporate a way to deal with noisy data since it enable the simultaneous matrix of rotation and translation. Finally, representing the FSA parameters with respect to the mean optimal screw axis is perhaps a way to normalize and compare different population group kinematics. This study shows that using the dispersion as mean of a representation of knee kinematics and the DQ algorithm, a repeatable and consistent information can be found from in vitro cadaver study.

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