Estimation of Whole Body Moment of Inertia Using Self-imposed Oscillations

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***Abstract*— Able-bodied segment moment of inertia (MOI) can be determined from anthropometric tables and different models. These methods cannot be used for special populations such as obese subjects, elderly individuals, etc. The objectives of this study are to estimate the personalized in vivo moment of inertia of the whole body using three-dimensional (3-D) inverse dynamic and angular momentum approaches and compare their results to five other methods (Hanavan's model, photogrammetry, two anthropometric tables, and inverse pendulum) tested in seven individuals. With respect to the modified Zatsiorsky method, the inverse dynamic approach values were within 3.7% of those along the all principal axes while the other four methods were off by 3.4%. The angular momentum method values were within 2.0% of the modified Zatsiorsky method for all three axes of rotation. It appears that the proposed methods could be applied to estimate the personalized in vivo moment of inertia of people with different body morphology.**

Keyword- Moment of inertia, inverse pendulum model, inverse dynamic, angular momentum, photogrammetry

I. INTRODUCTION

Body segment moment of inertia values are required to estimate the joint muscle moments during clinical evaluations, sport activities, ergonomic tasks, etc. to identify the underlying mechanical determinants that facilitate or compromise mobility. Most often these values are estimated from anthropometric tables obtained from cadaver populations. They do not represent accurately the majority of the adolescent population or the non-abled individuals. Data from children could not be used to estimate those in a developing spinal deformity.

The segments MOI can be determined by various non-clinical techniques. The pendulum technique (direct and inverted) has been used to calculate the moment of inertia in cadavers and applied by Brenière [2]. This method required the calculation of a constant that included the mass, the moment of inertia of the whole body and the distance of the COM to the ground. Though the moments of inertia can be calculated in the antero-posterior and transverse axes, the model does not permit its estimation along the vertical axis. A more comprehensive clinical method to estimate the personalized 3-D mass moment of inertia is needed. Moiré techniques and other video-based surface topography methods are used for measuring body surfaces as well as the changes when subjected to deformation, stress, etc. These are particularly suited for back surface measurements, especially for spinal deformity. They meanwhile provide only a rough estimate of the moment of inertia of the trunk because of the uncertainty in the physiological parameters (density values for bones, muscles and other soft tissues). Other methods such computerized axial tomography are relatively accurate to estimate the moment of inertia but can not be considered since they are costly, time-consuming and invasive for a regular clinical follow-up.

The objectives of this study were to estimate the personalized in vivo moment of inertia of the whole body using three-dimensional (3-D) inverse dynamic and angular momentum approaches and compare their results to five other methods (Hanavan's model [5], photogrammetry [6], two anthropometric tables [3], [4], inverse pendulum [2]) tested in seven individuals. Fig.1 shows the three-dimensional inverted pendulum model [1].

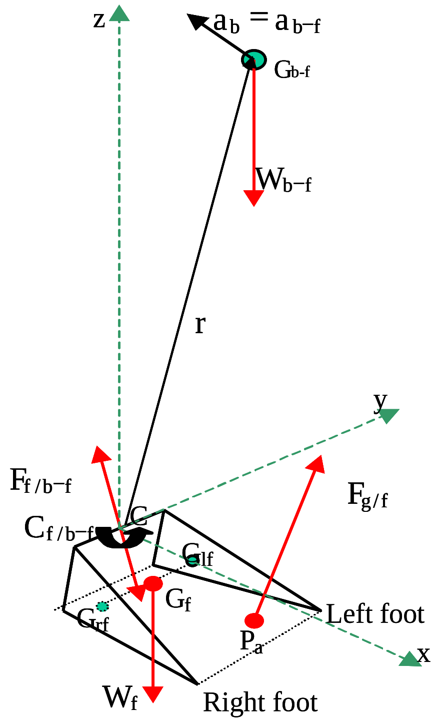

Figure 1. Inverse pendulum representation of the COM of the human body oscillating about a point lying midway between the ankles (Barbier, 2003)

II. THREE- DIMENSIONAL INVERTED PENDULUM MODEL OF THE COM

The proposed method consists of tracking body segments during self-imposed rotations about the ankles by means of a high resolution video-based system of six cameras and force-plate data. Two analytical avenues are to be explored. The first one deals with equations of motion and calls for a 3-D inverse dynamic model to described the self-imposed oscillations of an inverse pendulum. The second avenue applies the 3-D angular momentum equations to the body segments estimated from video data and the angular impulse calculated from the force-plate data.

Our 3-D inverse dynamic pendulum model was used to estimate the trajectory of the COM (Barbier et al., 2003). In this paper we have shown that the body moves as a rigid body during both quiet standing posture and self-imposed oscillations. Using this model, it is possible to estimate the moment of inertia of the body (excluding the feet) when the subject oscillates about the ankles. Normally, the muscle moments and the joint reaction forces are normally the unknown values in an inverse dynamic calculation. But by substitution, the muscle moments can be eliminated and another parameter can be chosen as an unknown, the moment of inertia in this case. These are articulated at point C representing the center of rotation at the ankle level shown in Fig. 1. It is located at the midpoint between the two malleoli. The position of point C in the summation of moments does not affect the COM excursion since the moment transfer terms will be taken in the summation of moments of the COM of the body less the feet. Consequently, setting the origin between the ankles does not exclude conditions involving individuals who usually do not stand with 50% of their weight on each foot. Euler's equation of motion is applied to the fixed foot segment to estimate the ankle moment at point C

$$\mathbf{C}_{\mathbf{b\text{-}f/f}} + \mathbf{M}_{\mathbf{Fg/b}} + \mathbf{M}_{\mathbf{wf}} = 0 \tag{1}$$

where $\mathbf{C}_{\mathbf{b\text{-}f/f}}$, $\mathbf{M}_{\mathbf{Fg/b}}$, and $\mathbf{M}_{\mathbf{wf}}$ are the ankle muscle moments, reaction force moments, and weight of the feet moment, respectively. By rearranging (1), the ankle muscle moments can be expressed as

$$\mathbf{C}_{\mathbf{b\text{-}f/f}} = -\mathbf{M}_{\mathbf{Fg/b}} - \mathbf{M}_{\mathbf{wf}} \tag{2}$$

Afterwards, body moments except feet are taken at the ankle.

$$\mathbf{C}_{\mathbf{f/b\text{-}f}} + \mathbf{M}_{\mathbf{wf\text{-}f}} = \mathbf{I}\,\boldsymbol{\alpha} \tag{3}$$

where $\mathbf{M}_{\mathbf{wf\text{-}f}}$, $\mathbf{I}$ and $\boldsymbol{\alpha}$ are weight body less the feet moment, moment of inertia tensor, and angular acceleration of the body less the feet, respectively. Since $\mathbf{C}_{\mathbf{b\text{-}f/f}} = -\mathbf{C}_{\mathbf{f/b\text{-}f}}$ and combining the above equations the unknown muscle moment is eliminated.

$$\mathbf{M}_{\mathbf{Fg/b}} + \mathbf{M}_{\mathbf{wf}} + \mathbf{M}_{\mathbf{wf\text{-}f}} = \mathbf{I}\,\boldsymbol{\alpha} \tag{4}$$

In (4), all the parameters are known except **I**. The reaction forces are obtained from force-plate data; the weight of the feet is determined form the weight of the body using conventional anthropometric tables; body weight is obtained by weighing and the lever arms are given by markers located on the feet and body measured by an six cameras video-based kinematics system and anthropometric tables.

The second model based on the three dimensional inverse pendulum model calls for the angular momentum equation. The first step is to calculate the whole body COM linear velocity vector **(v)**. Since the distance between the origin and the COM of the body less the feet is assumed to be constant and of length *r*, and mass of the body less the feet (m) is calculated by anthropometric tables [4], the angular momentum of the whole body less the feet about the ankle $(\mathbf{H}_o)$ is calculated by

$$\mathbf{H}_o = \mathbf{r} * (m * \mathbf{v}) \tag{5}$$

since

$$\mathbf{H}_o = \mathbf{I}_o * \boldsymbol{\omega} \tag{6}$$

where $\boldsymbol{\omega}$ is the angular velocity of the whole body COM, and $\mathbf{I}_o$ is the whole body less the feet MOI about the ankle.

III. METHODS

Seven able-bodied male subjects participated in the experiments. Their average age and weight were 32.29±5.25 years and 85.81±10.79 kg, respectively. Their height was 1.80±0.06m. Subjects had no previous orthopedic ailment or neurological disorders that could affect his standing posture and the three self-imposed oscillations.

Seven methods were applied to estimate the moment of inertia of the whole body. These were 1) the truncated cone model (Hanavan, 1964), 2) photogrammetry method [6], 3) cadaver (Dempster, 1955) and 4) generalized in-vivo anthropometry (de Leva, 1996), 5) personalized in-vivo model (Brenière, 1996), 6) inverse dynamic, and 7) angular momentum procedures.

Hanavan's model (1964) is the often used method to estimate body segments inertias in three-dimensions. Twenty-five anthropometric measurements of the subjects are needed to estimate the mass moment of inertia of each body segment. The photogrammetric method is based on the assumptions that the body is composed of elliptical zones of two centimeters in width and that the segments densities are known. The dimensions of the zones were obtained by digitizing photographic records taken simultaneously from the lateral and frontal views of the subjects.

Anthropometric tables derived from cadaver studies and in-vivo investigations have been applied widely to estimate the segments moment of inertia. In the study, the subjects' whole body MOI about the center of mass was estimated using data from Dempster (1955) and de Leva (1996).

Brenière's method [2] provides personalized in-vivo inertial parameters. To obtain them, the subjects were asked to oscillate back and forth about their ankles in the antero-posterior (AP) and medio-lateral (ML) axes respectively. For each type of oscillations a constant that includes the moment of inertia was calculated. This method can not be applied for the vertical axis.

The proposed inverse dynamic and angular momentum methods give also personalized in-vivo inertias but for each axis. The subjects were fitted with eight 2.5-cm markers to define the feet and whole body COM. The COM of the feet was estimated by putting four reflective markers at the right and the left fifth metatarsophalageal joints and both lateral malleoli. The vertical position of the COM in upright standing was calculated by the reaction board method. The AP and ML positions of the whole body COM were measured by the force plate. The subject stands at about the center of the force plate in quiet standing position, while the feet were parallel to each other with a distance of 100 mm from the center of the force plate. The mean value of the COP coordinates was considered as the position of whole body COM with respect to the center of the force plate. Then, four reflective markers were put over the trunk at the height of the COM aligned to it on front, back and both sides of the trunk. These markers were used for tracking the COM of the whole body during different trials.

Six video cameras were located around the force plate at a distance of about 3 m from its center. During an acquisition, video and force plate data were collected simultaneously at 60 Hz for a of 20-second period. Afterwards, video and force plate data were filtered with a fourth-order zero-phase lag Butterworth filter having a cutoff frequency of 6 Hz to reduce the noise.

The subject was evaluated for three experimental conditions. The first was a self-imposed AP oscillation of about 15 degrees corresponding approximately 10 cm for 20s followed by a self-imposed ML oscillation performed in the same range of the AP oscillations. The third experiment was a rotation of 20° about the vertical axis of the body. There were five trials for each condition and the mean value was used to calculate MOI. Particular care was given that the trunk and lower limbs oscillate in unison to avoid double-pendulum motions where the hips and shoulders move in opposite directions. Equation (4) and (6) were applied to estimate the moment of inertia of the whole body less the feet with respect to the ankle and then transposed to the body COM using the parallel axis theorem.

IV. RESULTS

Table I represents the estimated MOI of the subjects for the seven methods applied in the study. The moment of inertia average value about AP axis at the whole body COM was 14.85±2.16 $kg.m^2$. The lowest value was estimated by the angular momentum method (14.00±2.03 $kg.m^2$) while the highest was from the Brenière model (17.86±4.56 $kg.m^2$). The MOI estimated based on Hanavan's model (14.28±1.90 $kg.m^2$), de Leva anthropometric table (14.28±1.78 $kg.m^2$), and photogrammetry (14.10±1.93 $kg.m^2$) were close to the mean MOI value. The nearest MOI to the average value about AP was estimated based on inverse dynamic method (14.59±3.92 $kg.m^2$). This value was produced while the subjects' COM acceleration were located in a given range (e.g., while the first subject's COM acceleration had an average 0.82 rad/s^2 (ranging from 0.43 to 1.33 rad/s^2).

The average value about ML axis based on all the methods was 13.65±2.13 $kg.m^2$. The lowest value was obtained with the photogrammetry method (13.17±1.82 $kg.m^2$) while the highest was from the Brenière model (14.46±3.67 $kg.m^2$). The MOI values based on Hanavan's model (13.56±1.93 $kg.m^2$), Dempster's anthropometric table (13.57 ±1.79 $kg.m^2$), de Leva anthropometric table (13.46 ±2.02 $kg.m^2$), inverse dynamic method (13.72±3.31 $kg.m^2$), and angular momentum procedure (13.54±1.97 $kg.m^2$) were close to the mean value.

In the self-imposed oscillation about vertical axis, the MOI average value was 1.44±0.27 $kg.m^2$ at the subject's COM. The lowest value was estimated by angular momentum method (1.36±0.29 $kg.m^2$) while the highest one was based on the inverse dynamic method (1.51±0.77 $kg.m^2$). This value based on Hanavan's model (1.42±0.28 $kg.m^2$), photogrammetry (1.45±0.29 $kg.m^2$), and de Leva anthropometric table (1.41±0.26 $kg.m^2$) was considerably close to the average value.

There was no statistical difference between the MOI estimated by the different methods applied in the study.

V. DISCUSSION

The objectives of this pilot study were to develop a 3-D inverted pendulum model to calculate the personalized in-vivo moment of inertia of the human body and to compare these values to those obtained by five given methods along AP, ML, and vertical axes. These methods were Hanavan's geometrical model [5], photogrammetry method [6], anthropometric tables (de Leva [3], and Dempster [4]), and Brenière's method [2].

Both methods based on the inverse dynamic and angular momentum approaches gave values in the same order of magnitude as the other classical methods. Since the MOI estimations based on de Leva (1996) anthropometric table were among the highest and lowest ones and closer to the average of the subject' moment of inertia through the COM along all the three axes, it can be considered as a means of comparison with the others. For such a case, the values obtained with the angular momentum were only 2.0% off compared to the other methods (3.4% or more). The advantage of the angular momentum method over the de Leva (1996) technique resides in individualized in vivo values rather than estimations obtained from tables. Furthermore, this technique can easily be applied in special populations such as elderly persons, obese individuals and other groups for which tables are not available. In addition, with respect to the modified Zatsiorsky method [3], the inverse dynamic approach values were within 3.7% of those along the all three principal axes while the other methods were off 3.4%. The Pearson coefficient of correlation between the MOI estimated by inverse dynamic and angular momentum approaches and

TABLE I. AVERAGE VALUES OF SUBJECTS' WHOLE BODY MOI (KG.M^2) AT THE CENTER OF MASS FOR DIFFERENT METHODS.

Methods **Axis**	**Truncated cone model**	**Photographic ellipsoidal model**	**Cadaver or generalized in-vivo anthropometry**		**Personalized in-vivo model**		
	Hanavan (1964)		***Dempster (1955)***	***de Leva (1996)***	***Brenière (1996) (inverse pendulum)***	***inverse dynamic***	***angular momentum***
AP	14.28± 1.90	14.10± 1.93	———	14.28± 1.78	17.86± 4.56	14.59± 3.92	14.00± 2.03
ML	13.56± 1.93	13.17± 1.82	13.57± 1.79	13.46± 2.02	14.46± 3.67	13.72± 3.31	13.54± 1.97
Lon	1.42± 0.28	1.45± 0.29	———	1.41± 0.26	———	1.51± 0.77	1.36± 0.29

those estimated by de Leva anthropometric table [3] for each axis are given Table II. These are well over 0.9.

VI. CONCLUSION

Two personalized methods based on a three-dimensional inverse pendulum model were developed to estimate of the in vivo moment of inertia of the whole body using three self-imposed oscillations. The inverse dynamic method yielded the moment of inertia along all three principal axes within the range of the other methods. The same situation was met by the angular momentum procedure. Both methods were able to provide a good estimation of the MOI values compared with mean values estimated by all the methods applied in the study. For all three testing conditions, the coefficients of correlation of the MOI estimations were high between the both models and the de Leva [3] anthropometric table. It appears that the proposed methods could be applied to estimate the personalized in vivo moment of inertia of people with different body morphology.

TABLE II. PEARSON COEFFICIENT OF CORRELATION OF THE MOI ESTIMATED BETWEEN THE DE LEVA ANTHROPOMETRIC TABLE AND THE MODELS. DATA FOR THE THREE TEST CONDITIONS ALONG EACH AXIS

	de Leva (reference) anthropometric table		
Axis **Methods**	***AP (antero-posterior)***	***ML (medio-lateral)***	***Lon (longitudinal)***
Inverse dynamic	0.97 (0.00)	0.97 (0.00)	0.97 (0.00)
Angular momentum	0.97 (0.00)	0.99 (0.00)	0.94 (0.00)
Photogrammetry	0.98 (0.00)	0.99 (0.00)	0.98 (0.00)
Hanavan	0.99 (0.00)	0.99 (0.00)	0.98 (0.00)
Brenière	0.71 (0.00)	0.92 (0.00)	———
Dempster	———	0.97 (0.00)	———

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