Identifying Screw Motion from Noisy Data

[1]Koon Kiat Teu and [2]Wangdo Kim
[1]Computational Structure Mechanics
Institute of High Performance Computing, Singapore
[2]Department of Mechanical and Material Engineering, Portland State University, Portland, OR, USA
Email: mwdkim@gmail.com

Abstract—**Conventional techniques use point transformation to calculate screw axis. In this paper, we present a novel technique that directly estimates the axis of a screw motion as a Plücker line. The mapping of line vectors, which is made possible by the use of the dual transformation matrix, enabled skin movement artifacts to be significantly reduced. This method is compared with Halvorsen, et al. ["A new method for estimating the axis of rotation and the center of rotation." Journal of Biomechanics, vol 32, 1999. pp. 1221-1227] in simulations of random measurement errors and systematic skin movements. A clinical application on an ankle joint is shown to illustrate the method.**

Keywords-Screw axis; dual transformation; joint motion; kinematics; skin movement artefacts

I. INTRODUCTION

Screw axis methods have been employed to describe joint kinematics mainly in clinical assessments. However, screw axes have been difficult to obtain as its directions and positions are highly sensitive to data noise. The screw axes directions will deviate and its positions scatter in a seemingly random manner under noisy situations causing the result to be meaningless. Screw axis is also undefined with the normal set of equations when rotation angles equal zero, i.e. under a pure translation. Another set of equations is needed for this special case [1]. This caused additional difficulty in determining the screw axis under small rotations. Conventional algorithms generally used a set of point patterns with least square methods to solve the noise problem. Finding the axis of rotation usually involves mapping a set of data points to the corresponding data points after movement. Contrary to conventional algorithms, the new algorithm presented in this paper takes advantage of a set of line patterns that are derived from point data using a dual number relationship. The method involves mapping a set of vectors to the corresponding vectors after movement. The mapping of vectors is made possible by the use of the dual transformation matrix (DTM). The method can be easily adapted in clinical settings. To illustrate this, the method was applied to ankle data in an attempt to calculate the screw axis during ankle movement.

II. METHOD

A. Calcultaing the Screw Axis

The background on obtaining DTM is given in [2]. With the DTM obtained, the screw axis positions and directions can then be determined. Following the derivation by [3] and incorporating dual vector algebra in it, we show that the Plücker coordinates in dual vector representation, $\hat{V} = V + \varepsilon W$ of this screw axis satisfy the condition that it is an invariant of the 3x3 DTM $[\hat{R}]$. We write $[I - \hat{R}]\hat{V} = 0$ and seek a solution other than $\hat{V} = 0$. This is easily done if we separate it into the pair of vector equations comprising the primary and dual component, respectively,

$$\begin{aligned} [I - R]\boldsymbol{V} &= 0 \\ [I - R]\boldsymbol{W} &= [D]\boldsymbol{V} \end{aligned} \tag{1}$$

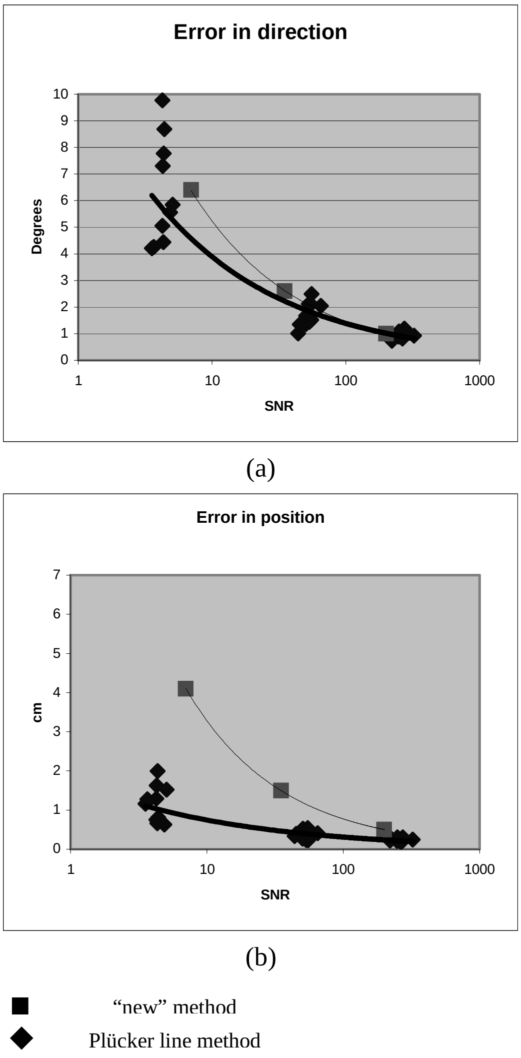
where [D] is the skew-symmetric matrix defined by $[D]\boldsymbol{V} = \boldsymbol{d} \times \boldsymbol{V}$ for any translation and we used the property of $[R]\boldsymbol{V} = \boldsymbol{V}$. Its algebraic derivation here is straightforward because it is a direct consequence of applying the eigen value problem to three-dimensional lines. It is probable that specifying the axis of a screw motion as a Plücker line is the most suitable linear representation because it minimizes the influence of non-linear transformations frequently used as closed form solutions in conventional methods.

In this paper, the approach for calculating the screw axis from DTM is stated by using the eigenvector and the singular value decomposition (SVD)[4]. It is adopted because it is more straightforward to implement and performs better under very small rotations. The first equation in (1) means that the $\boldsymbol{V}$, also the direction of the screw axis, is simply the eigenvector of the primary component, R, of the DTM. Looking at the dual component $\boldsymbol{W}$, the matrix $[I - R]$ turned out to be ill-conditioned and this can be efficiently evaluated using the SVD. The reference point **C** for the screw axis **V** is determined by $\boldsymbol{C} = \boldsymbol{V} \times \boldsymbol{W}$. More details of the method can be found in [5].

B. Verification of the Method

The proposed method was tested on simulated data. Two simulations closely following the method specified in [6] were carried out. In the first simulation, random sequences corresponding to measurement errors were added to the data set. In the second simulation, skin movement position artefacts were added with reference to [7]'s result. The skin movements were then added to the simulated data. Simulations with each set of errors varied at 50%, 100%, 150% and 200% were run.

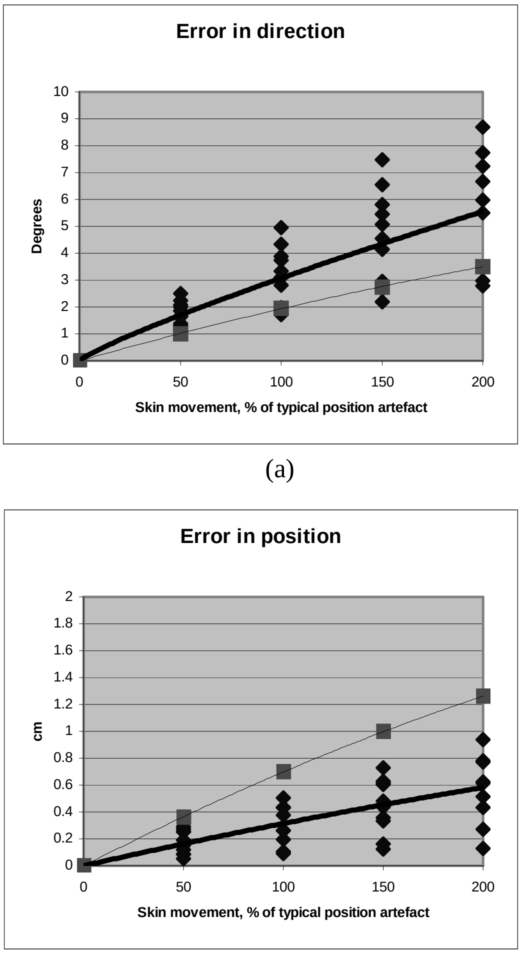
Comparison of the method to [8] was accomplished in [5]. The simulation results in [6] ("new" and "FHA" method) were used as references for the proposed algorithm. The results of the “new” methods were superimposed on our simulation results in Fig. 1 as a comparison. The general trend of the error follows that of those reference methods. Fig. 1 shows the results for the mean error in direction and position of the axis as a function of skin movement in the simulation. Polynomial lines of an order of 2 were also fitted to the simulated results using least squares fitting. The estimation of the position using the proposed method was better than both referenced methods. The error in direction follows closely to the “FHA” method in terms of magnitude and trend.



(a)

(b)

Figure 1. Mean error in the (a) estimated direction and (b) estimated position of the axis plotted against the SNR. The thick solid lines correspond to the presented method and the thin lines to the “new” method.



(a)

(b)

Figure 2. Mean error in the (a) estimated direction and (b) estimated position of the axis plotted against the magnitude of the skin position artefact. The thick solid lines correspond to the presented method and the thin lines to the “new” method.

Generally, the trend of the error follows that of those reference methods in this paper or [5]. The results show that when there’s little noise, all methods are comparable. However, as noise increases, the disparities in the methods become obvious. The error in direction for the Plücker line method was lower compared to the reference methods. The proposed method worked well for its applications, especially when noise is present. This is due to the fundamental difference between the proposed algorithm and the conventional methods. The mapping of vectors gives a better estimate of the rotation than mapping of points data, and hence better results could be derived from it. The results are significant as they demonstrate an advantage of using line instead of points. Together with the smoothing, the presented method is highly robust and provides reliable estimates of the screw axis, even under high noise situations.

III. APPLICATION

To test the method in a clinical setting, we calculated the screw axis with experimental data. The method was applied to the movement of the ankle joint which consisted of controlled movements.

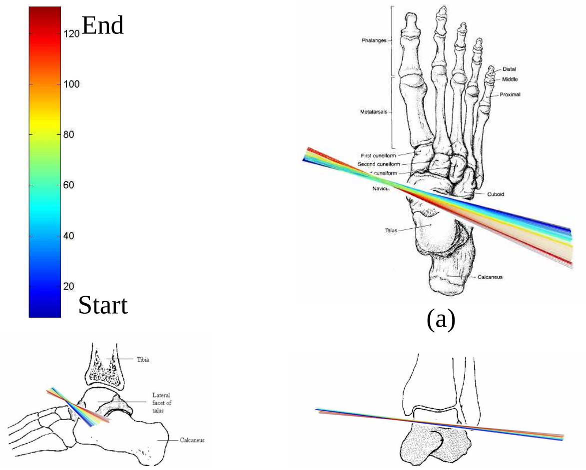
A. *Experimental Setup*

For the ankle joint movements, the data collected by in vitro experiments in [2] were used as the raw data. For details of the experimental setup, refer to [2]. The following gives a brief synopsis of the experiments. In the in vitro experiments, the passive motions at the ankle joint complex were measured by the ‘Flock of Birds’ electromagnetic tracking system. In the study, the passive motions during the dorsiflexion-plantarflexion and eversion-inversion were measured without considering weight and muscle forces. Ten below-knee amputation cadaver specimens were tested. Prior to the experiment, each specimen was thawed for 24 hours at room temperature and then dissected carefully to remove the soft tissue around the ankle joint complex, while the ligamentous system remained intact. The soft tissue at the head of the fibula and the tibia tuberosity were also removed. Four landmarks for definition of the tibia-fibula anatomical coordinate system were identified and marked for digitization. The screws of the receiver fixtures were inserted into the tibia, the talus, and the calcaneus.

After the setup was adjusted to the appropriate position, the specimen with three receivers attached was mounted to the setup. At the neutral position, each landmark was digitized five times. The position and orientation of the receivers attached to the bones were also recorded. After collecting the required data at the neutral position, while the footplate was rotated manually and slowly from the maximum dorsiflexion of the specimen to the maximum plantarflexion, the position and orientation of the receivers were recorded continuously. The same procedure was repeated ten times. The average of the results obtained from these measurements was used as the motion pattern during the dorsiflexion-plantarflexion for analyzing the joint kinematics of the specimen. Once the kinematic data during the dorsiflexion-plantarflexion were collected, the setup was rotated to simulate the movement of inversion-eversion. Similarly, while the footplate was rotated manually from the maximal inversion of the specimen to the maximal eversion, the kinematic data of the receivers on the bones were recorded continuously. Again, the motion of eversion-inversion was recorded ten times.

B. *Experimental Results*

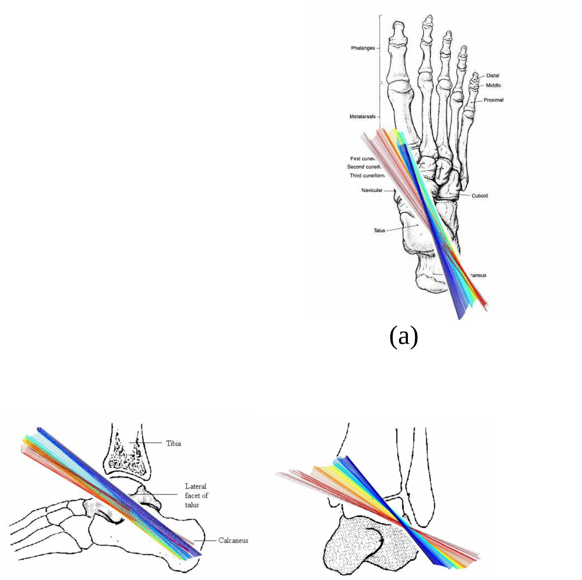
Fig. 3 and 4 shows the typical result of the screw axis movement of the ankle during the dorsiflexion-plantarflexion and inversion-eversion motion, respectively. In the graph, each instantaneous axis is plotted with a different color, starting from blue at the start of the movement to red at the end of the movement. The migration trend was clearly shown with the color-coded axes. The results are superimposed on anatomical drawings of the foot in three different views to show the position of the axes and magnitude of the migration in relation to the foot.



(a)

(b) (c)

Figure 3. (a) Top view, (b) side view and (c) posterior view of screw axes during dorsiflexion to plantarflexion, superimposed on the ankle joint. (Pictures adopted from [9] and [10])



(a)

(b) (c)

Figure 4. (a) Top view, (b) side view and (c) posterior view of screw axes during inversion to eversion, superimposed on the ankle joint.

IV. DISCUSSION

The axes are clearly not fixed, which is in agreement with studies on various joints. As can be seen from the graphs, at dorsiflexion, i.e. the beginning of the movement, the dorsiflexion is coupled with eversion as shown from the directions of the axes in Fig. 3a (top view). From Fig. 3c (posterior view), the dorsiflexion is shown to be coupled with abduction of the foot. At plantarflexion, i.e. the end of the movement, the plantarflexion is shown to be coupled with inversion (Fig. 3a) and adduction (Fig. 3c). Fig. 4 shows the typical result of the screw axis movement of the ankle during the inversion-eversion motion. Again, the migration trend was clearly shown in this case. As can be seen from the graphs, at inversion, i.e. the beginning of the movement, the inversion is coupled with plantarflexion as shown from the directions of the axes in Fig. 4a (top view). From Fig. 4b (side view) the inversion is shown to be coupled with adduction. At eversion, i.e. the end of the movement, the eversion is shown to be coupled with dorsiflexion (Fig. 4a) and abduction (Fig. 4b).

From these results we can see that the screw axis does not lie in the middle of the ankle joint. This implies that using a hinge or ball and socket joint would not be appropriate to obtain an accurate model of the ankle joint. Even if a hinge joint is used for simplification, it should be offset, e.g. to the front of the ankle for dorsiflexion-plantarflexion action, to give a better estimation. The above experiments were done with sampling frequency of 50Hz. The low sampling rate produces relatively good results. This shows that the algorithm works well in clinical settings and can be applied to normal actions where a sampling frequency of 50Hz suffices.

V. CONCLUSION

In our study, the estimation of screw axes from a set of Plücker lines clearly outperforms the conventional methods using points data. The presented method also showed that a screw axis could be easily determined from the DTM simply because the axis is invariant during the transformation. This opens the range of new practical applications of Plücker lines in bio-kinematics situations. The proposed eigenvector and SVD method also worked well and gives a simple alternative to the dual vector calculation. As illustrated, the algorithm also works well in clinical settings. The method provides an alternative analysis tool for estimation of screw axes and can be applied to any actions such as gait analysis in clinical settings to estimate the screw axis.

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