PROJECT COMPRESSION UNDER GENERALIZED PRECEDENCE RELATIONS

Salah Eldin Elmaghraby
North Carolina State University, Raleigh NC.

Seminar at UVHC, June 23, 2011

STATEMENT OF THE PROBLEM

HOW TO "COMPRESS" A PROJECT (SHORTEN ITS DURATION) AT MINIMAL COST UNDER GPR's?

BEGS THE FOLLOWING QUESTIONS:

1. **How to represent a project ?**

2. **What are GPR's?**

3. **What assumptions are made on individual activity time-cost relation?**

4. **How to do it?**

PROJECT REPRESENTATION:

GRAPHICAL REPRESENTATION:

AoA

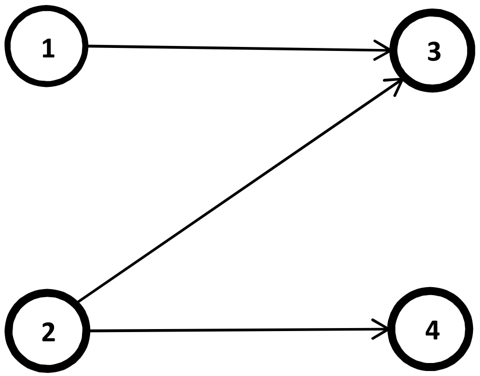
AoN

Gantt Chart

MATRIX REPRESENTATIONS:

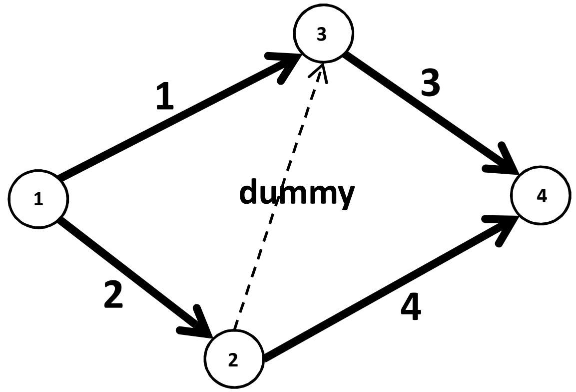
Incidence Matrix

Path-Link Matrix



AoN

1→3, 2→3,4



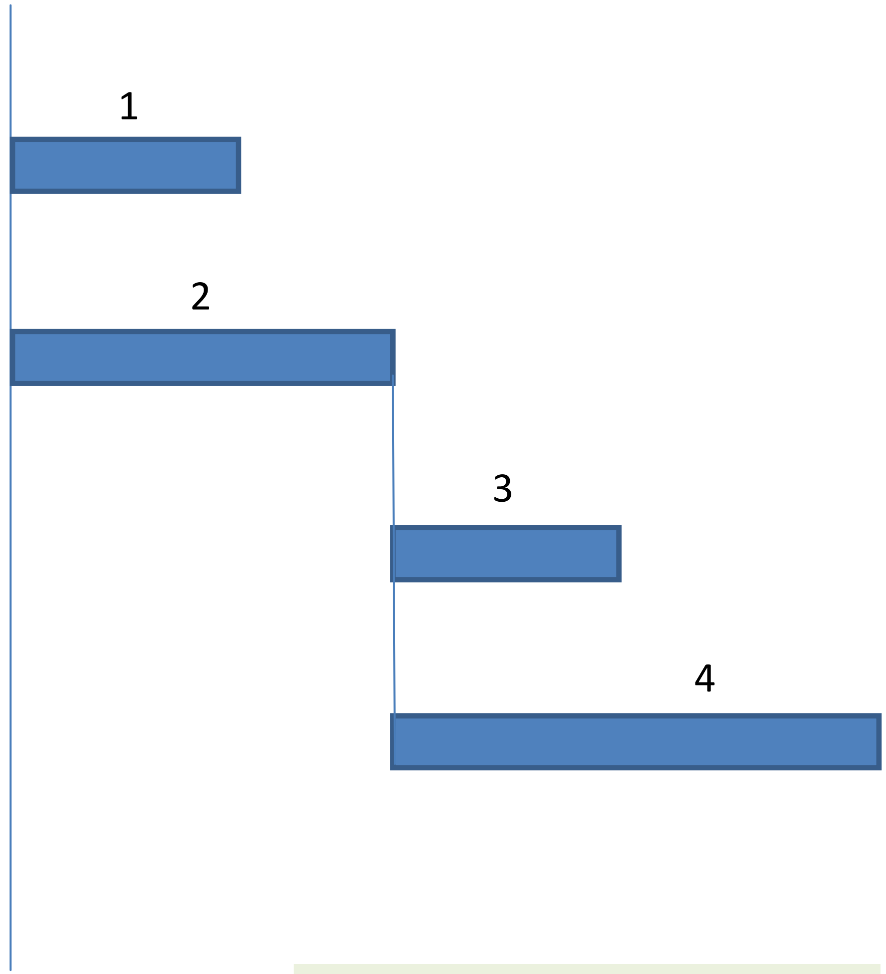
AoA

Act.	1	2	3	4
1			1	
2			1	1
3				1
4				

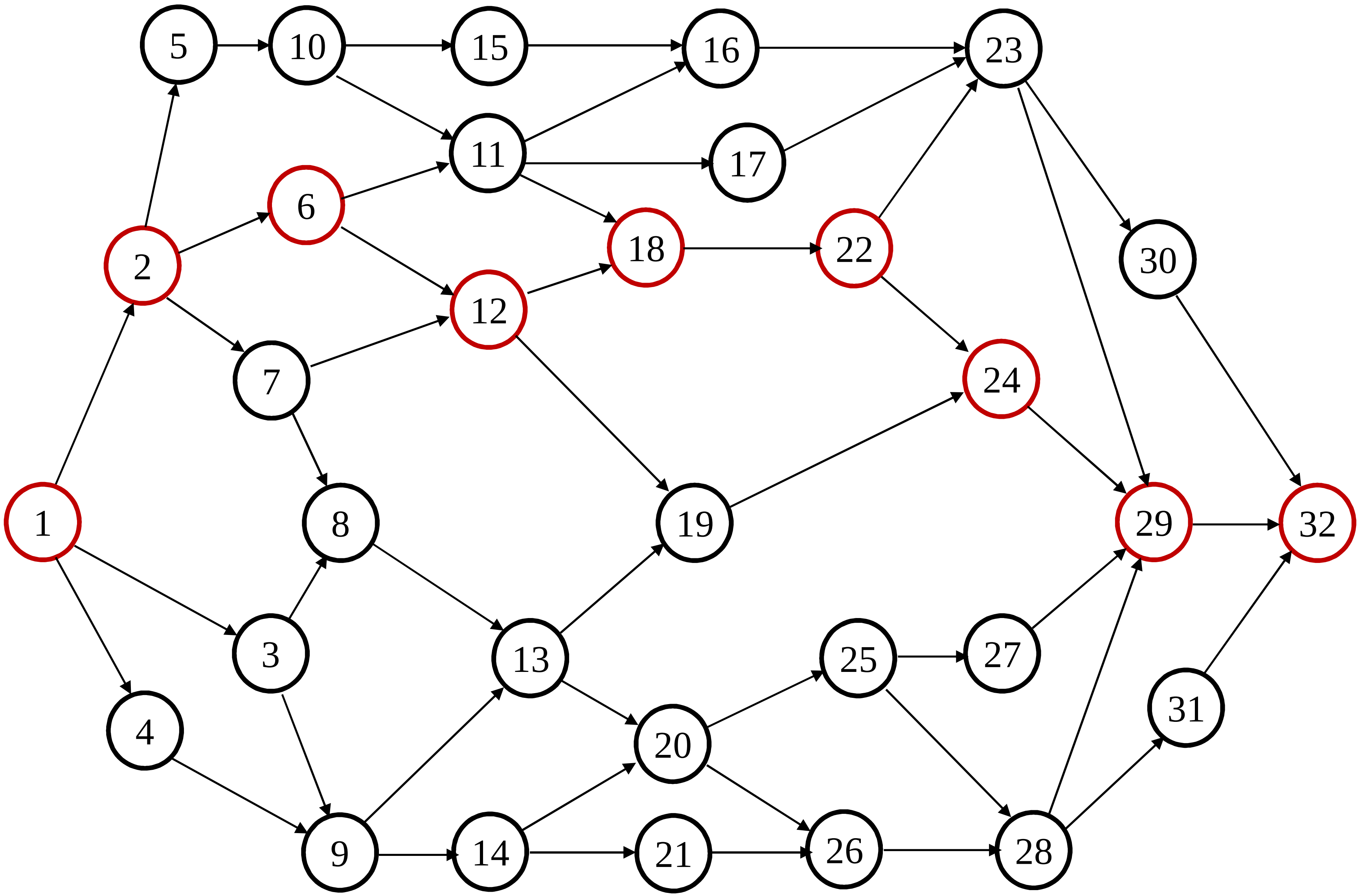
Incidence Matrix

Act. \ Path	{1,3}	{2,d,3}	{2,4}
1	1		
2		1	1
3		1	1
4			1

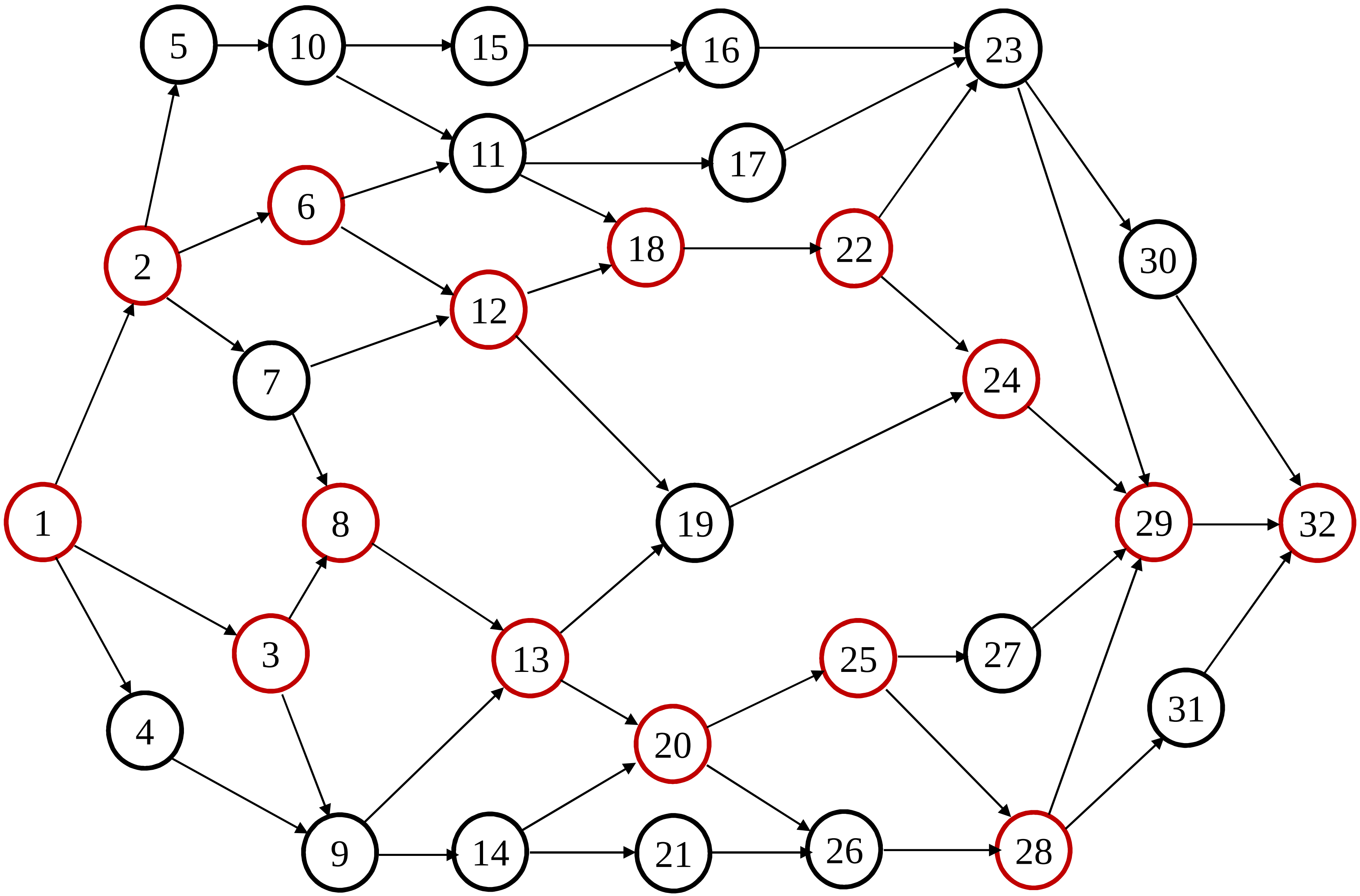
Path–activity matrix



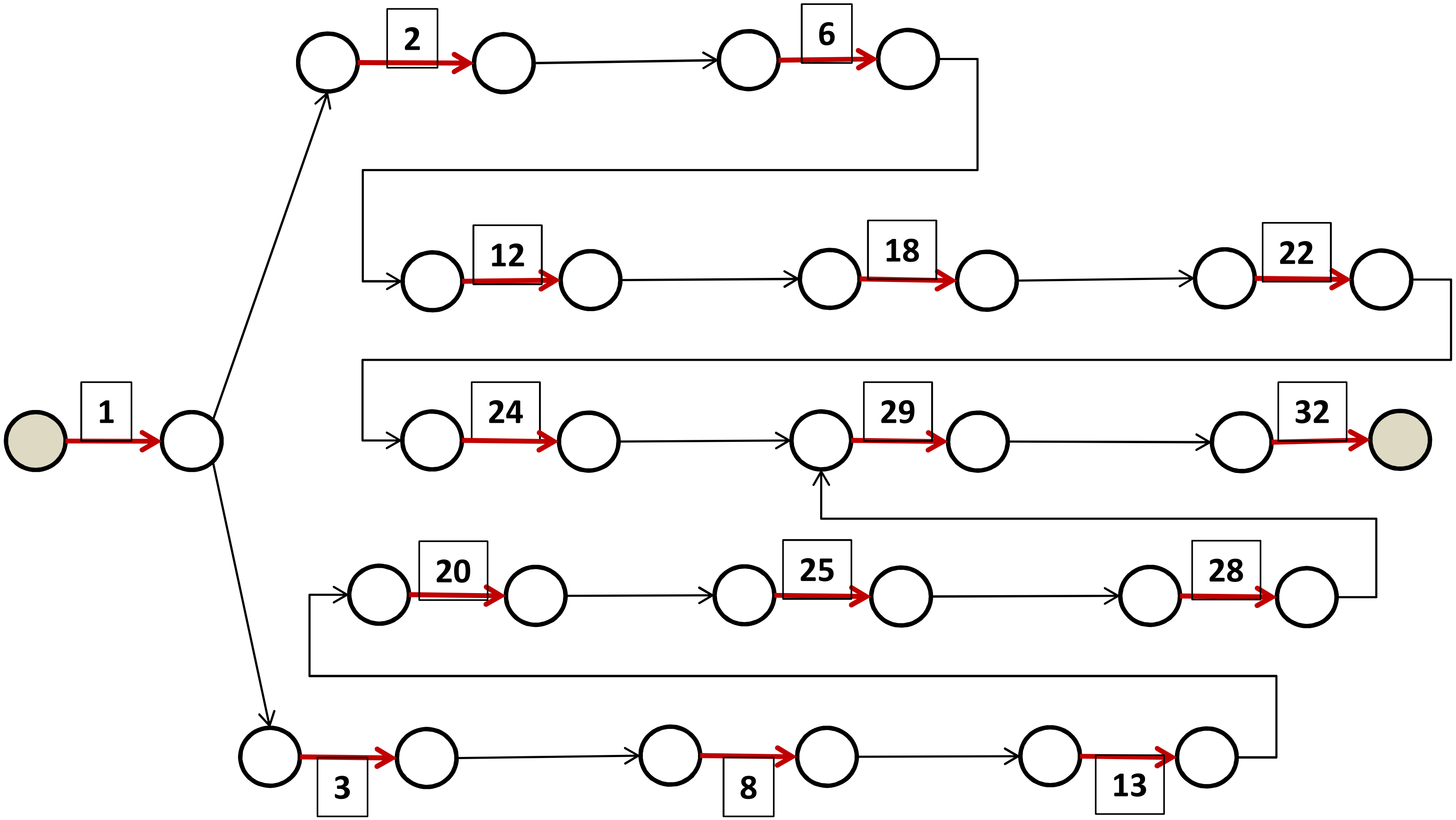
Gantt Chart of the IG



32-activity project – AoN -- a CP.



32-activity project – AoN -- a CS.



32-activity project – AoA -- a CS.

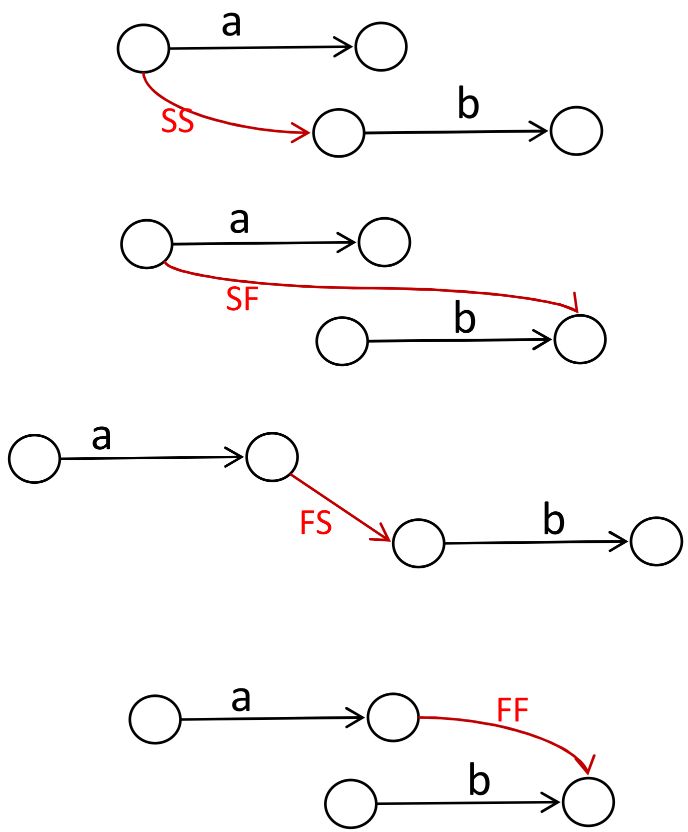
THE GPR's

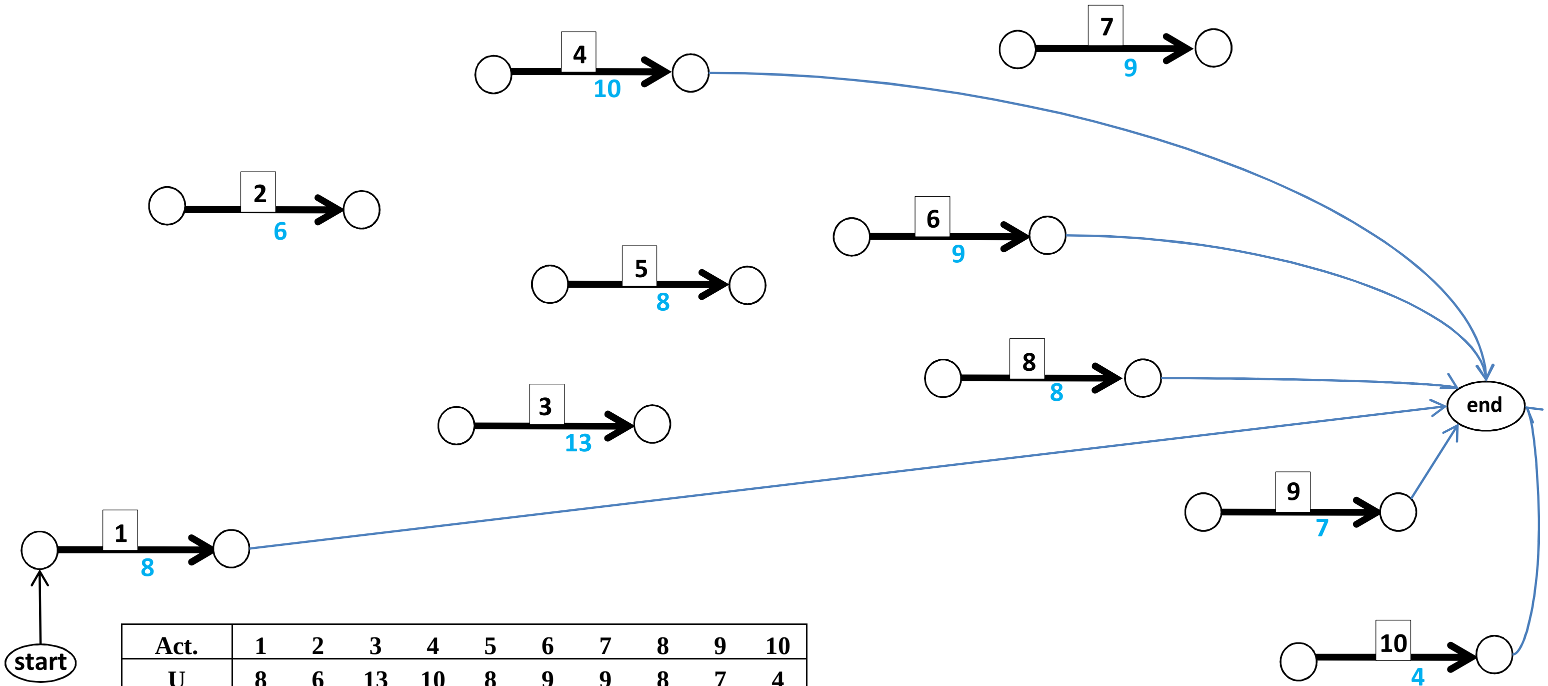
SS(a,b): ⟹ $s(b) \geq s(a) + \Delta$.

SF(a,b): ⟹ $f(b) \geq s(a) + \Delta$.

FS(a,b): ⟹ $s(b) \geq f(a) + \Delta$.

FF(a,b): ⟹ $f(b) \geq f(a) + \Delta$.

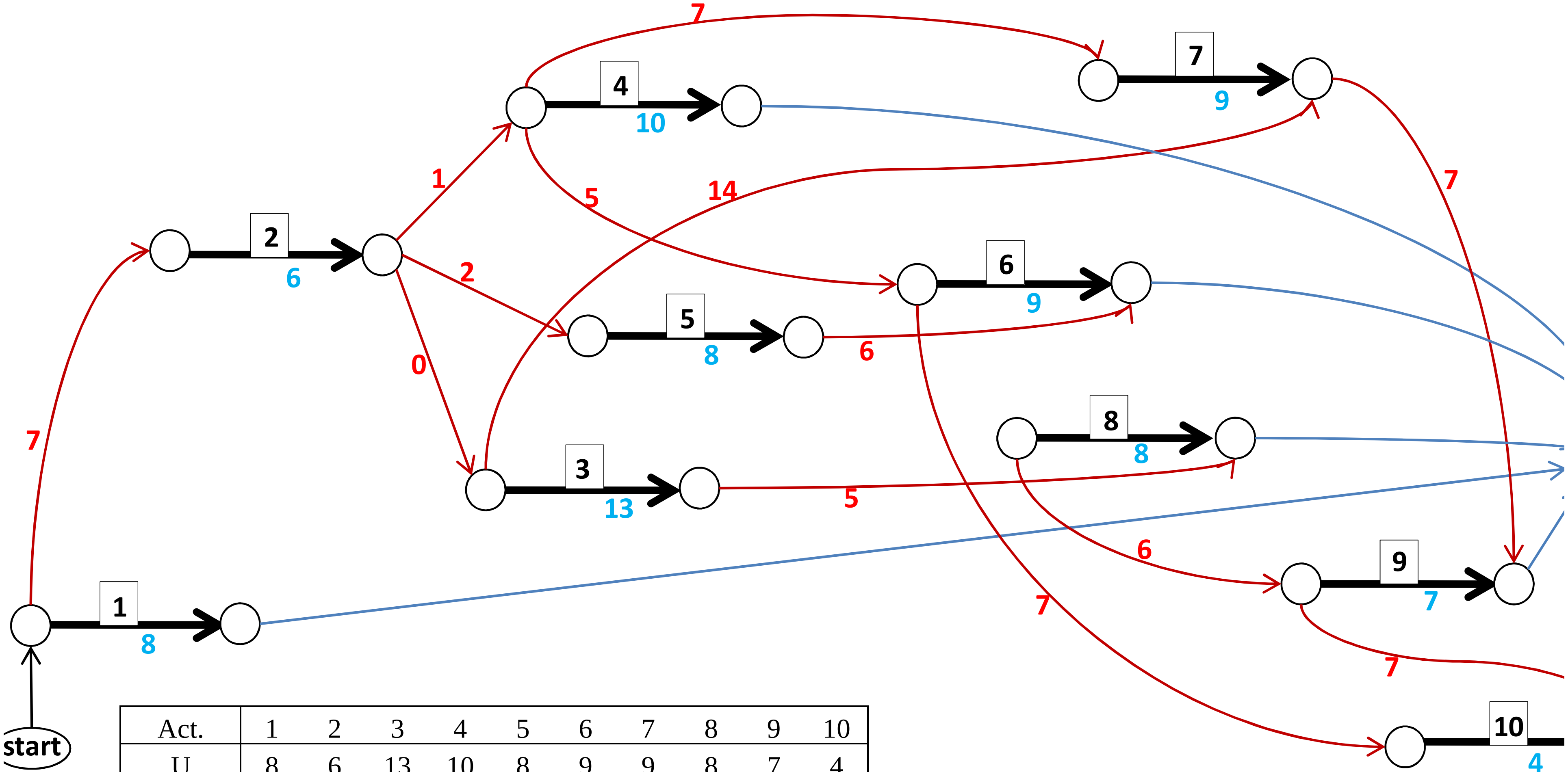




Act.	1	2	3	4	5	6	7	8	9	10
U	8	6	13	10	8	9	9	8	7	4
L	6	5	10	10	7	7	6	5	6	2
cost	10	13	16	∞	9	14	11	16	9	15

SS(1,2)=7
SF(3,7)=14, FF(3,8)=5
SS(4,6)= 5, SS(4,7)=7
FF(5,6)=6
SS(6,10)=7
FF(7,9)=7
SS(8,9)=6
SF(9,10)=7

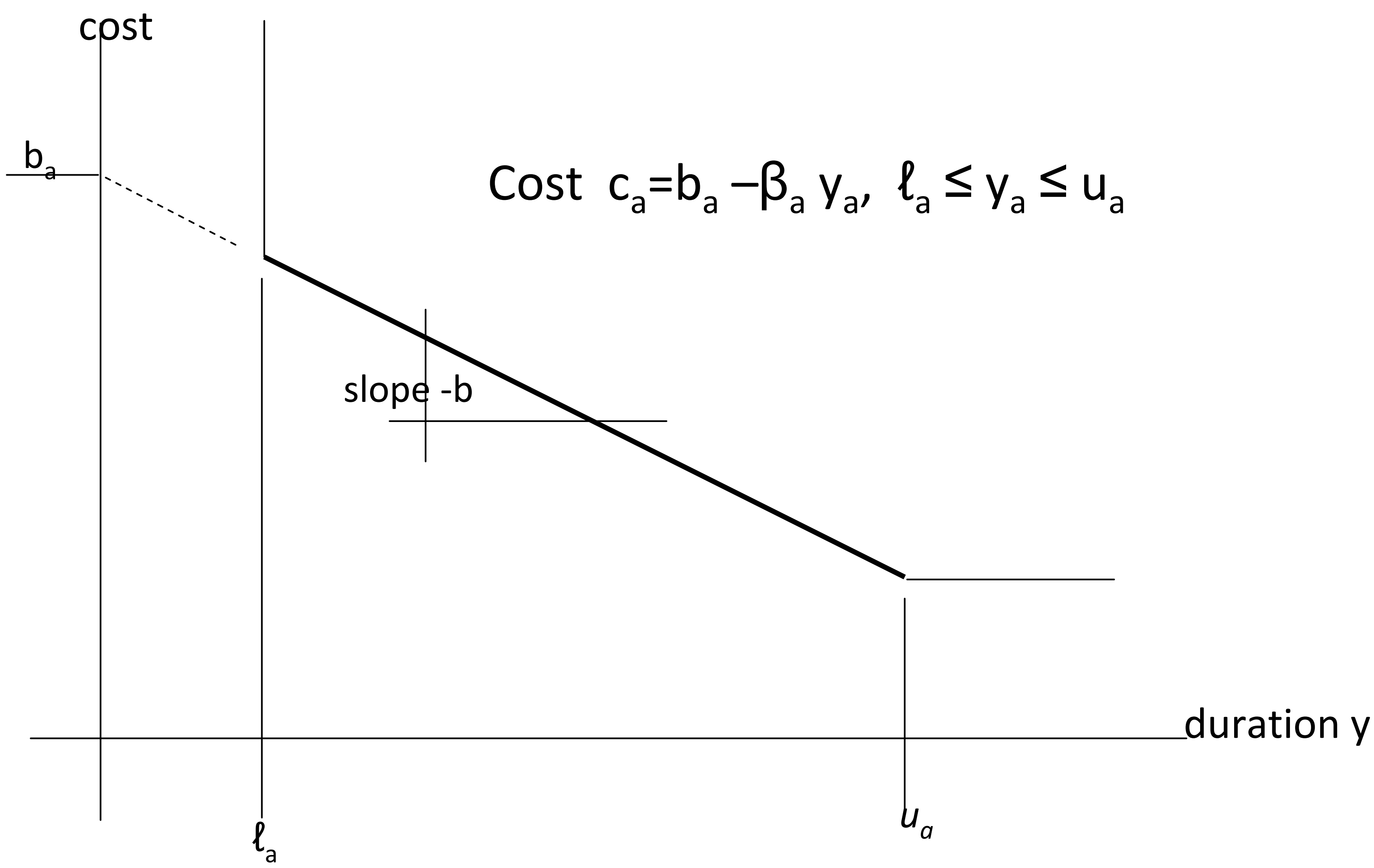
The original problem (10 activities) and its Data.

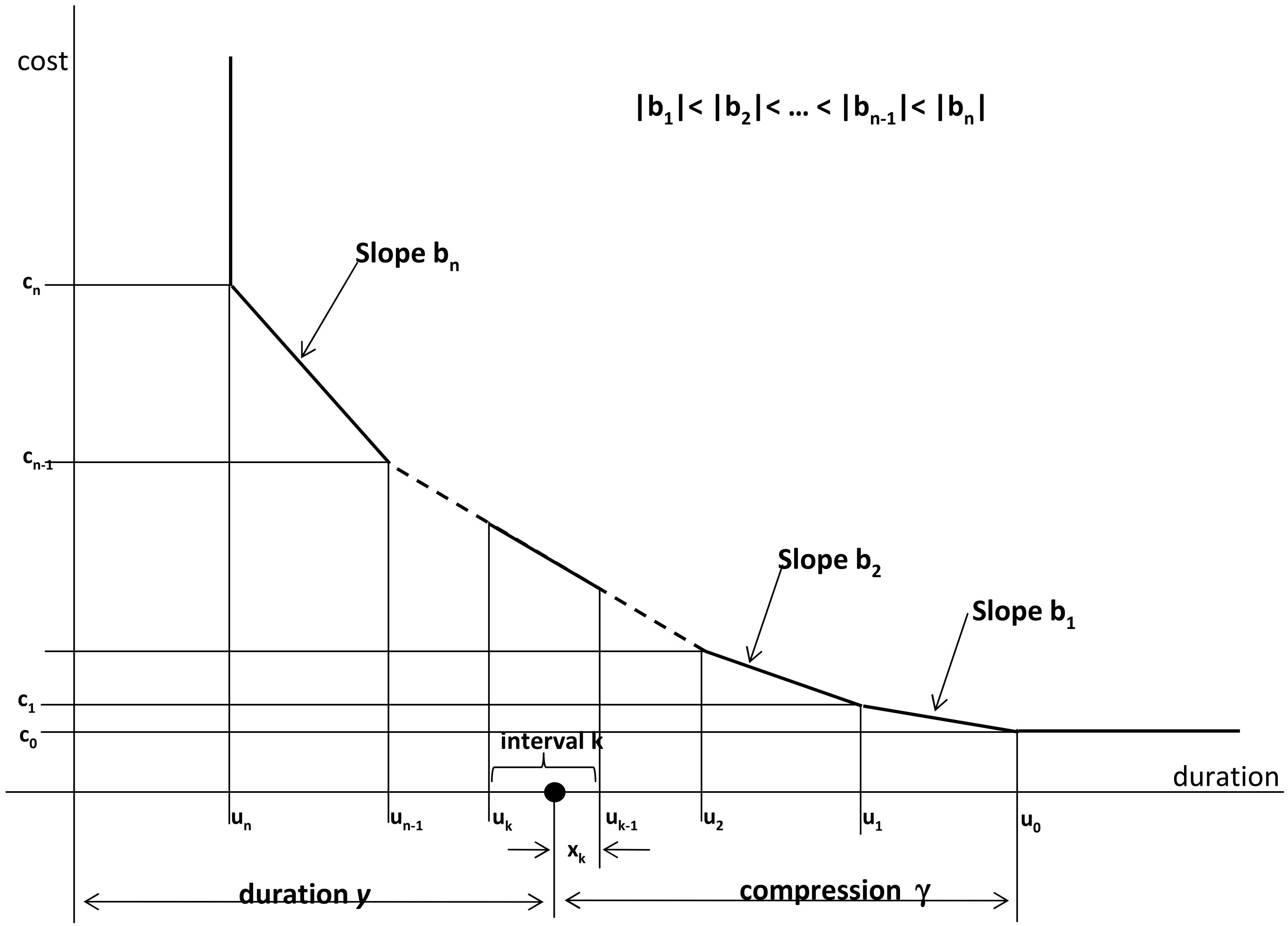


Act.	1	2	3	4	5	6	7	8	9	10
U	8	6	13	10	8	9	9	8	7	4
L	6	5	10	10	7	7	6	5	6	2
cost	10	13	16	∞	9	14	11	16	9	15

SS(1,2)=7
SF(3,7)=14, FF(3,8)=5
SS(4,6)= 5, SS(4,7)=7
FF(5,6)=6
SS(6,10)=7
FF(7,9)=7
SS(8,9)=6

The original problem and Data with GPR’s.

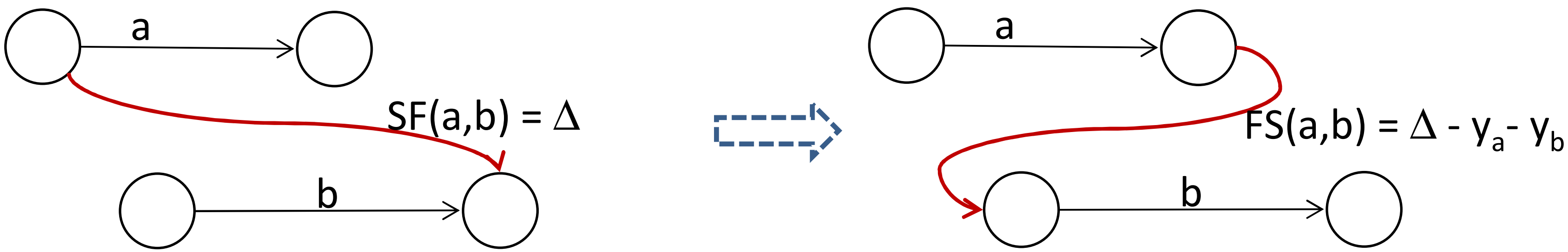

cost
b_a
Cost $c_a = b_a - \beta_a y_a$, $\ell_a \le y_a \le u_a$
slope -b
duration y
ℓ_a
u_a


cost
|b1|< |b2|< … < |bn-1|< |bn|
Slope bn
cn
cn-1
Slope b2
Slope b1
c1
c0
interval k
duration
un
un-1
uk
uk-1
u2
u1
u0
xk
duration y
compression γ

THE TRANSFORMATION OF FF- AND SF-RELATIONS INTO FS-RELATION

The original GPR was specified as FF.

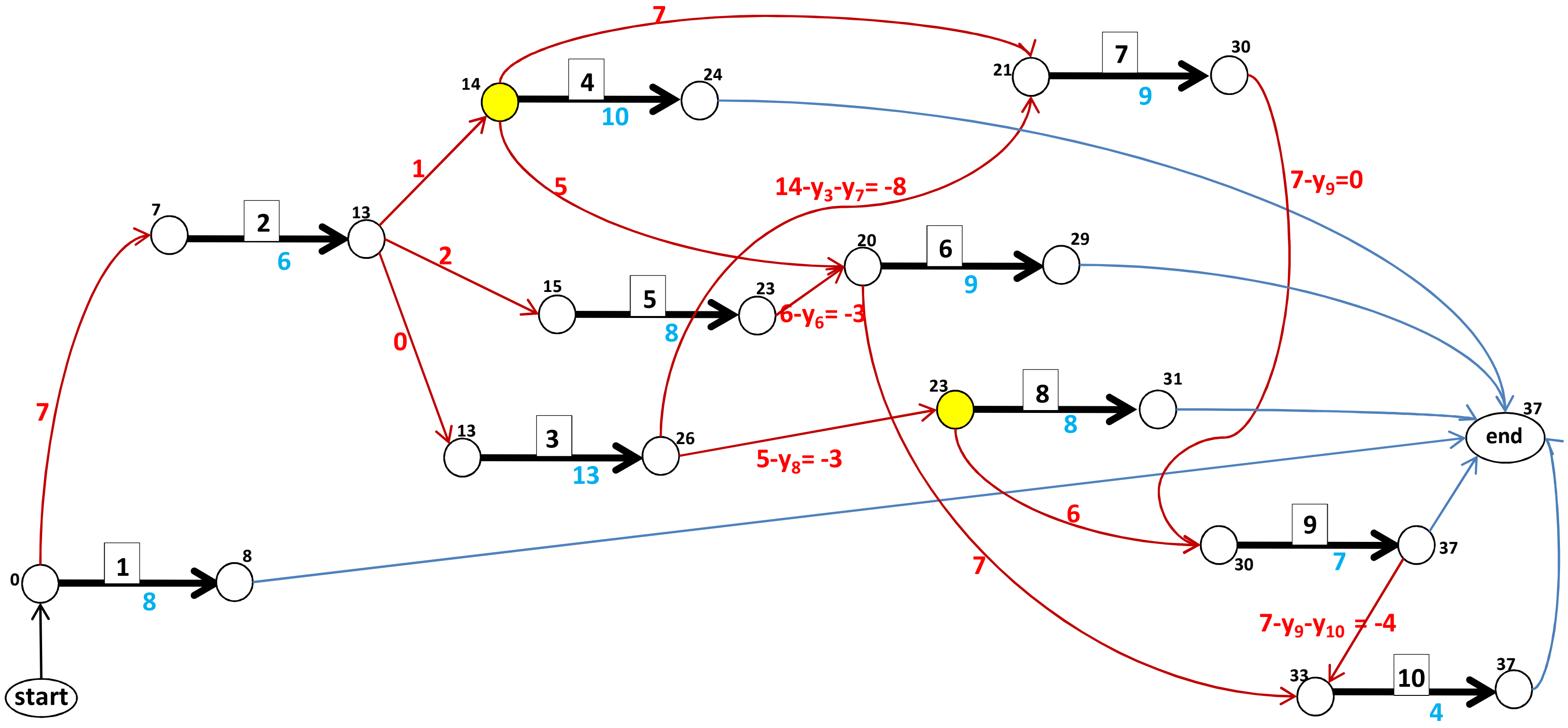
Transformed into FS-relation



The original GPR was specified as SF.

Transformed into FS-relation

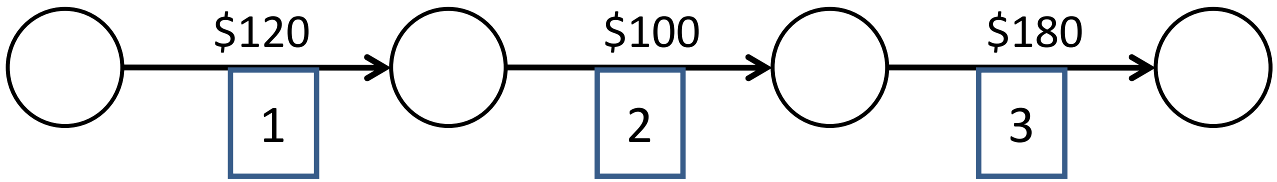
Activity *k* has s_k=2k-1 and f_k=2k.



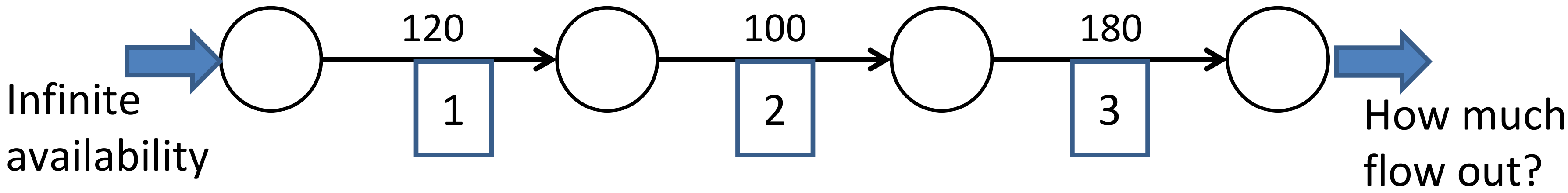
Act.	1	2	3	4	5	6	7	8	9	10
U	8	6	13	10	8	9	9	8	7	4
L	6	5	10	10	7	7	6	5	6	2
cost	10	13	16	∞	9	14	11	16	9	15

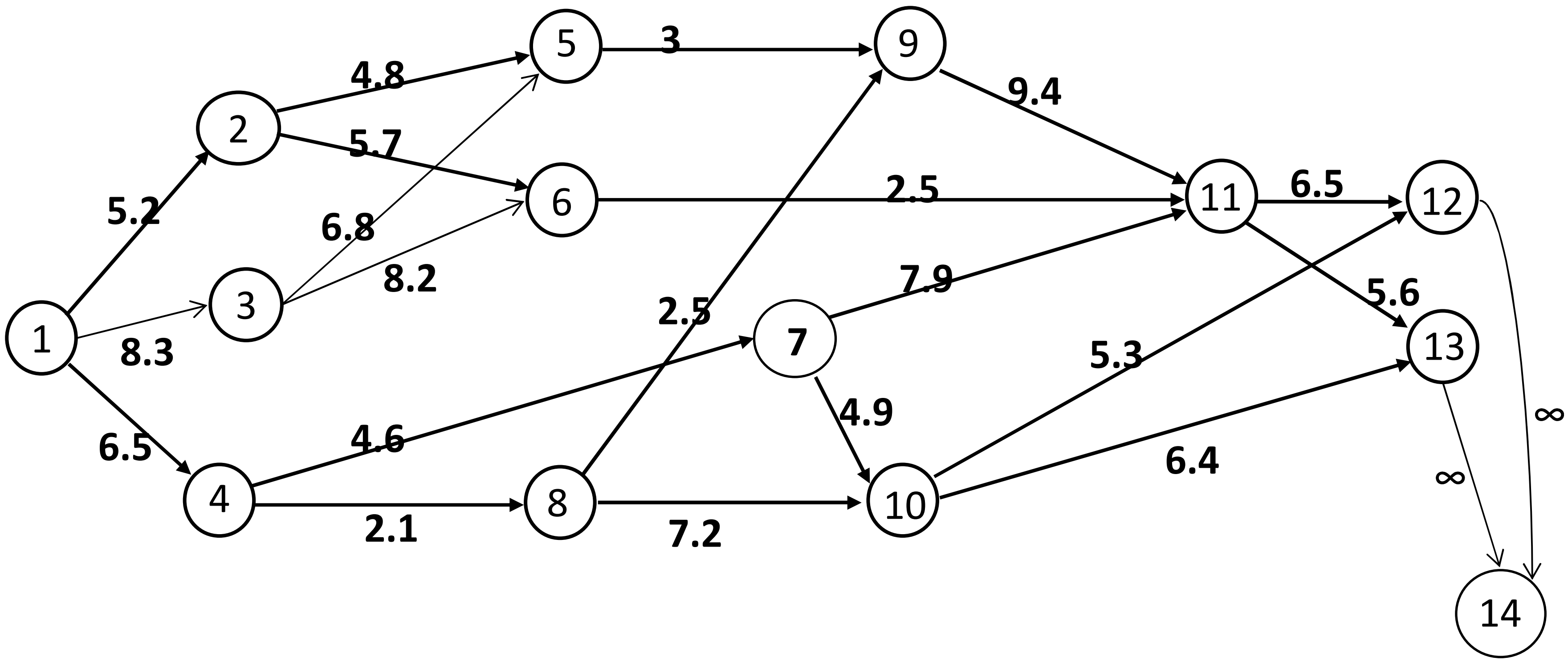
Original Problem after FS-transformation.

REMINDER:
HOW TO SECURE THE CHEAPEST SUBSET OF ACTIVITIES TO SHORTEN VIA FLOW ARGUMENTS.

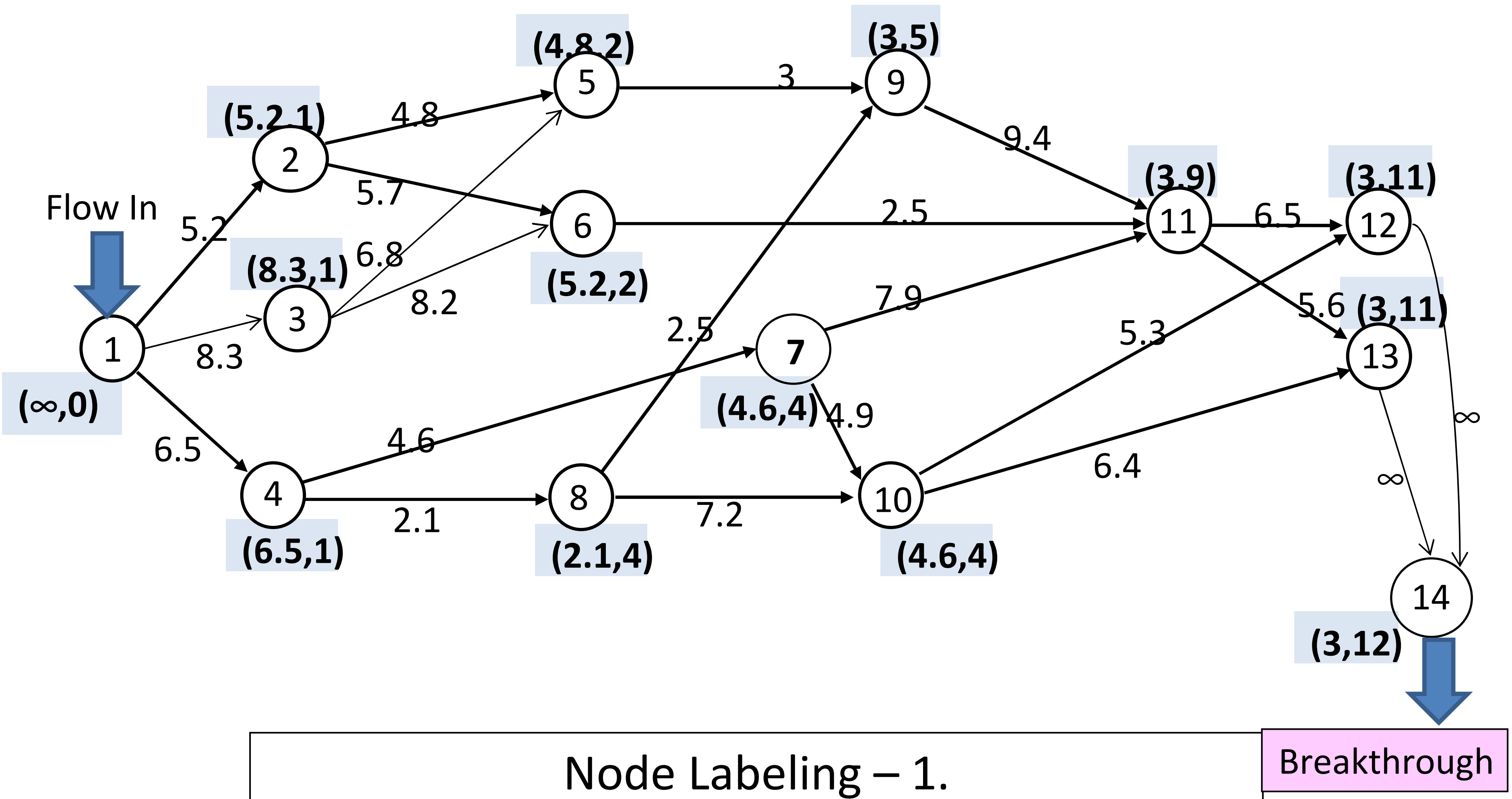


Interpretation of marginal cost as capacity.



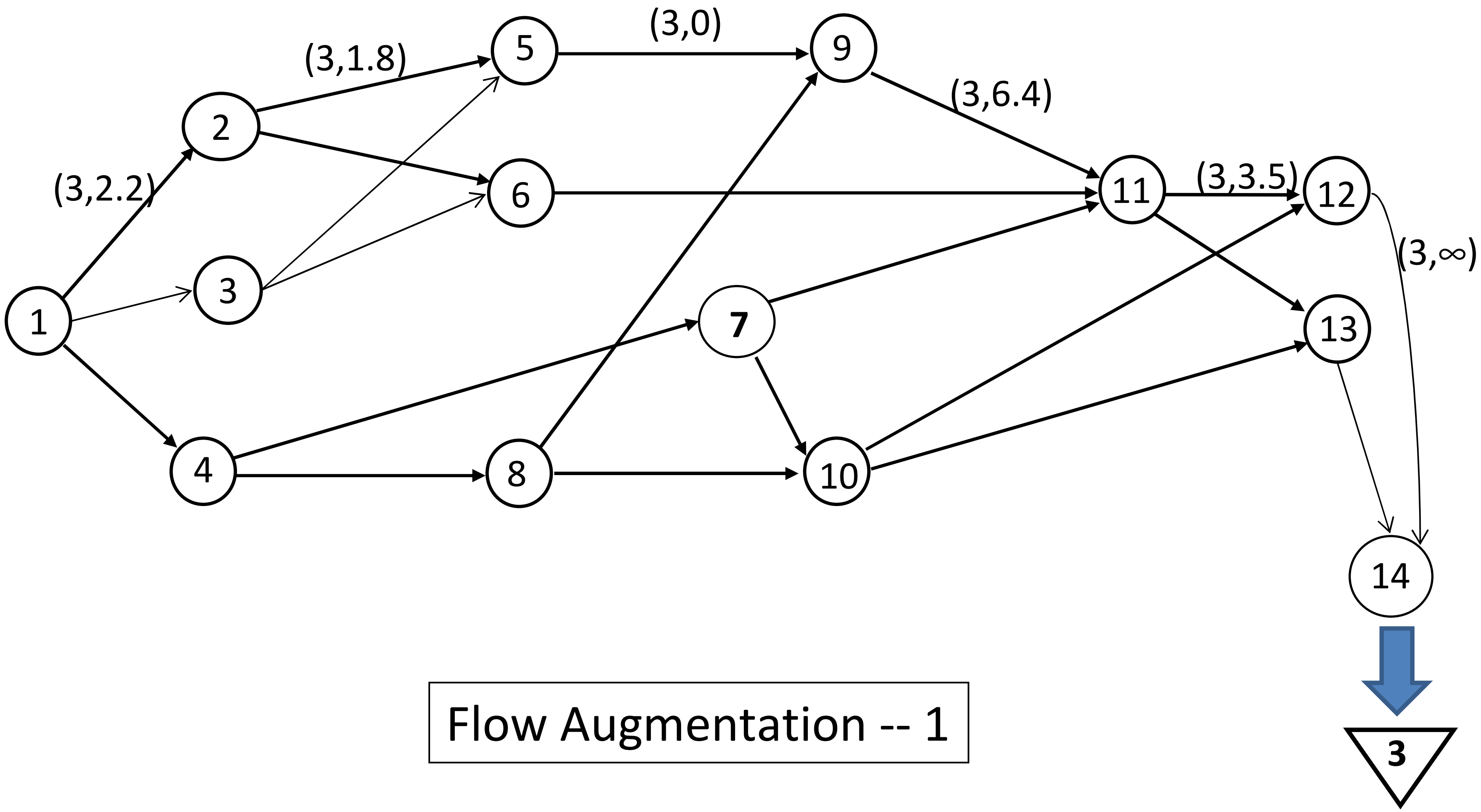


The number on the arc is the activity's marginal cost.

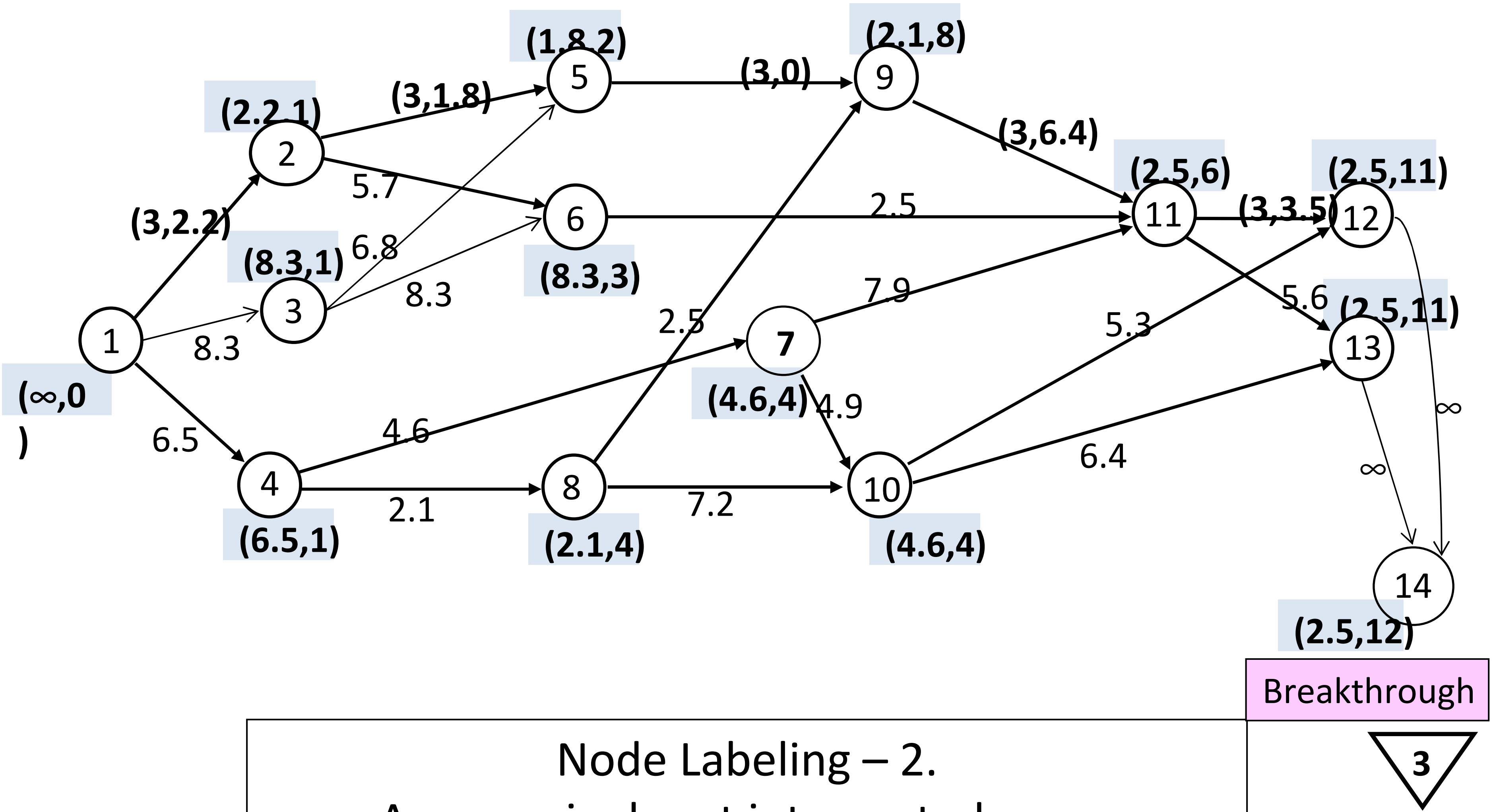


Node Labeling – 1.

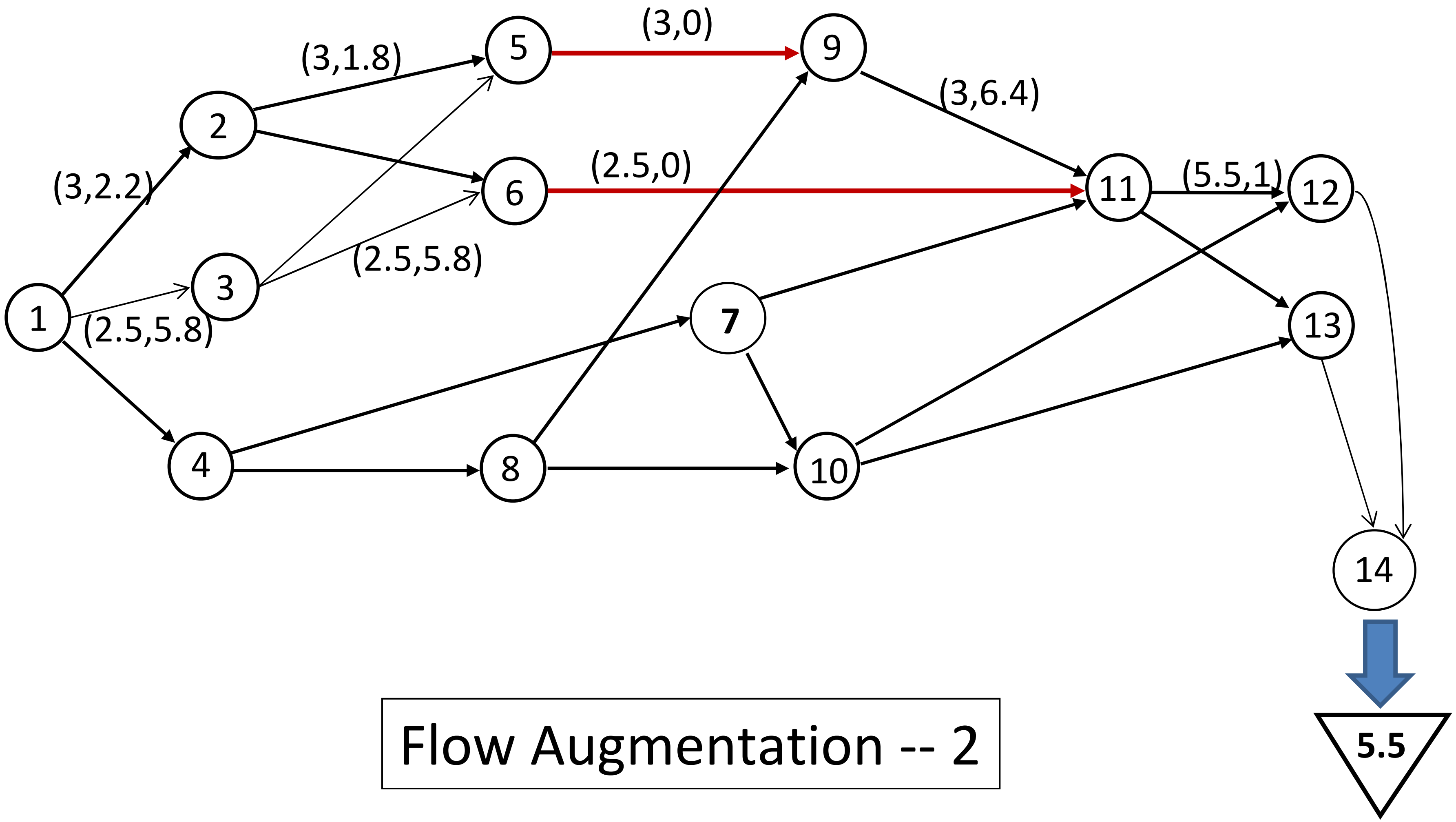
Arc marginal cost interpreted as arc "capacity".



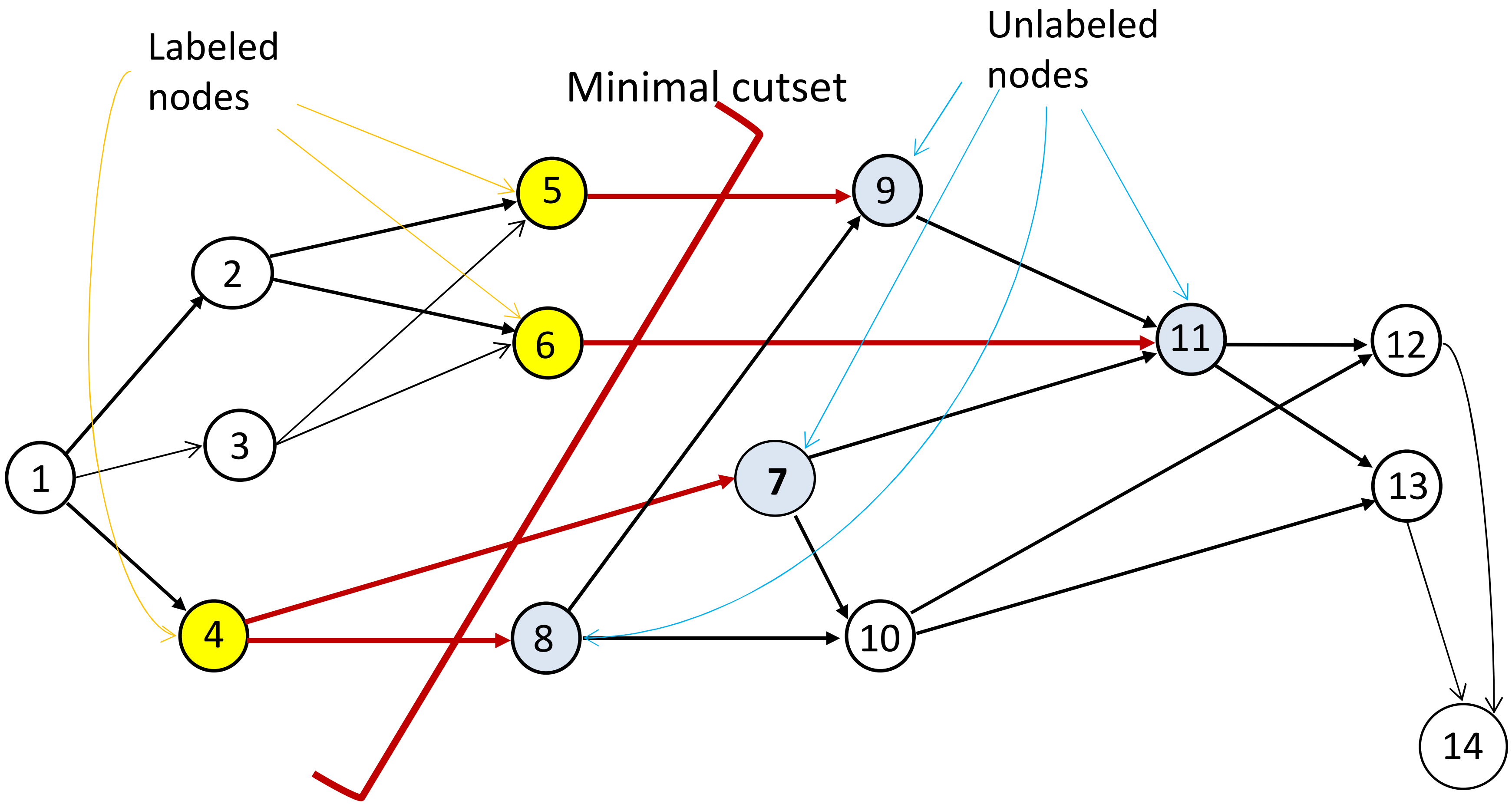
Flow Augmentation -- 1



Node Labeling – 2.
Arc marginal cost interpreted as arc "capacity".

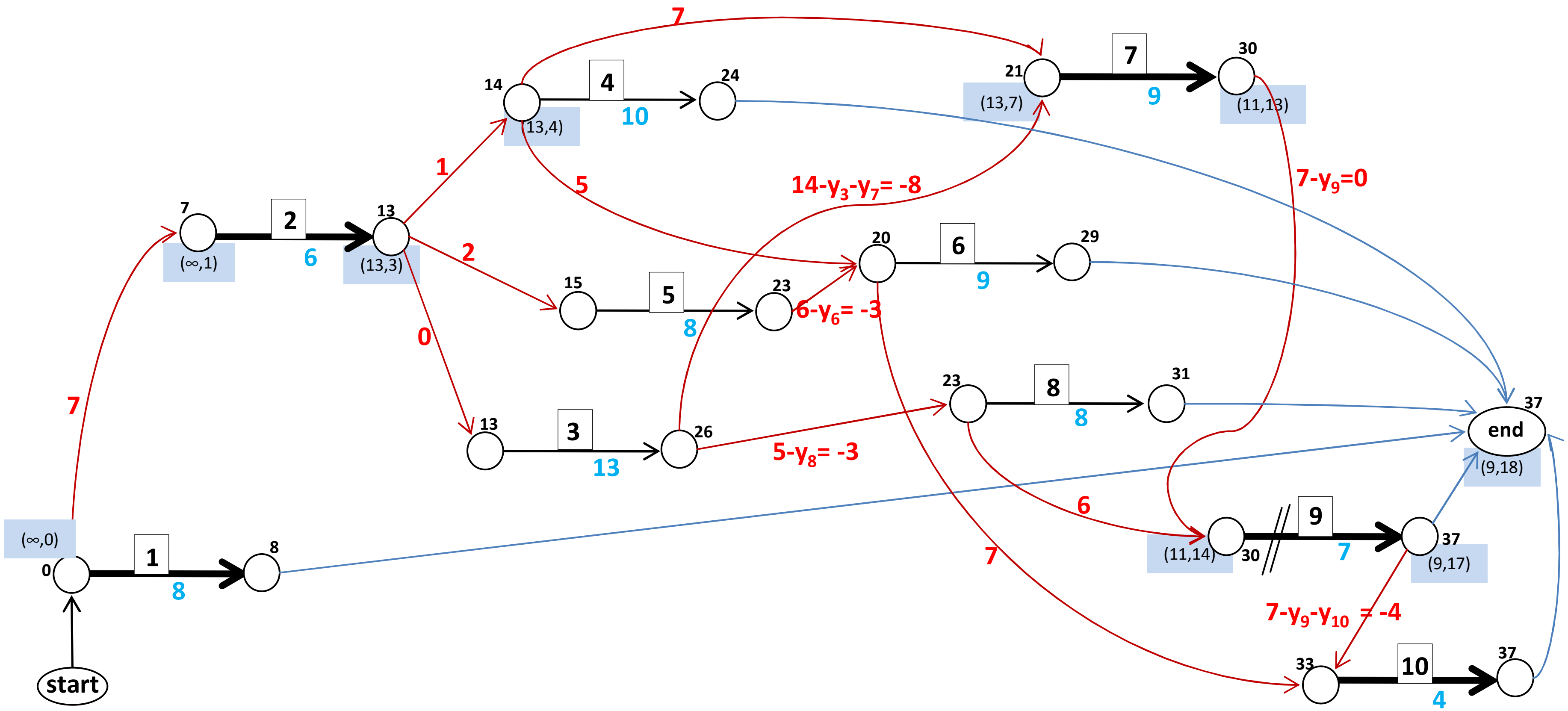


Flow Augmentation -- 2



The detection of the min-cost cutset via flow arguments.

Activity k has s_k=2k-1 and f_k=2k.



Act.	1	2	3	4	5	6	7	8	9	10
U	8	6	13	10	8	9	9	8	7	4
L	6	5	10	10	7	7	6	5	6	2
cost	10	13	16	∞	9	14	11	16	9	15

Breakthrough with f=9

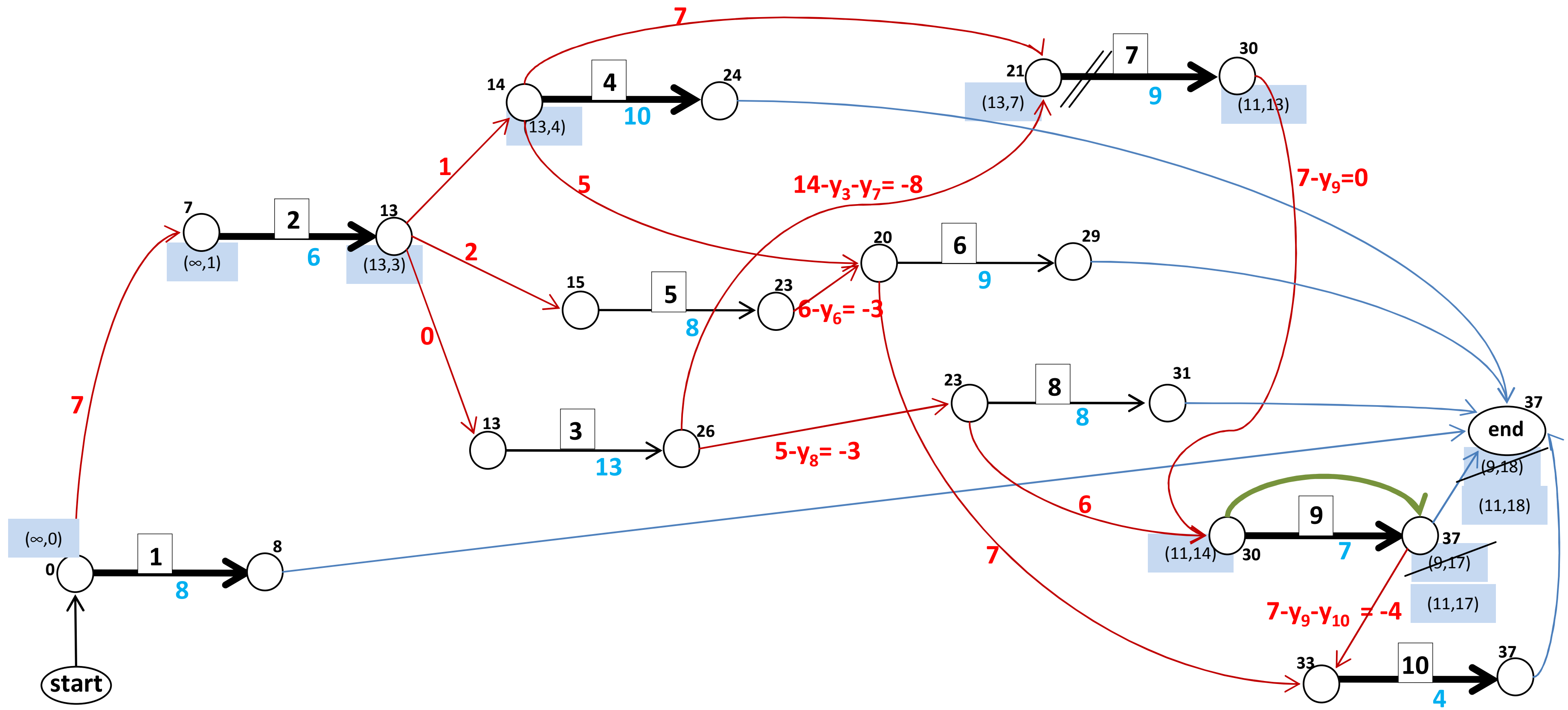
Iteration 1. Flow augmentation.

Arcs in red are GPR's. Arcs in blue are added to have a unique terminal.

CS = path {s→1→3 →4 →7→13→14→17→18→e}

∪ {18→19 →20 →e}.

Activity *k* has s_k=2k-1 and f_k=2k.



Act.	1	2	3	4	5	6	7	8	9	10
U	8	6	13	10	8	9	9	8	7	4
L	6	5	10	10	7	7	6	5	6	2
cost	10	13	16	∞	9	14	11	16	9	15

~~Breakthrough with f=9~~

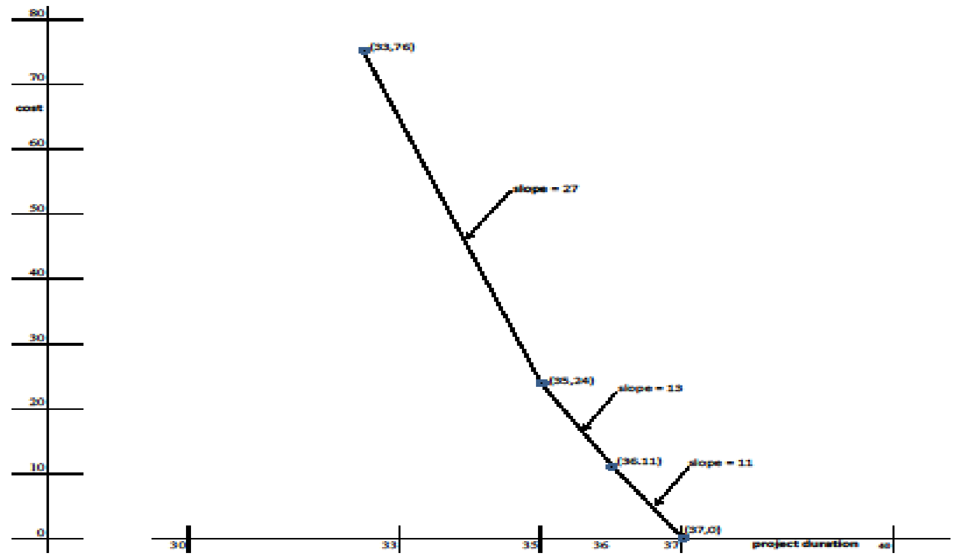
Breakthrough with f=11

Iteration 1. Flow augmentation.

Arcs in red are GPR's. Arcs in blue are added to have a unique terminal.

CS = path {s→1→3 →4 →7→13→14→17→18→e}

∪ {18→19 →20 → e}.



Time-Cost Function for Project Duration.

THE TIME-COST GRAPH.

WHERE TO GO FROM HERE?

1. Piece-wise linear and non-convex marginal costs.

2. Resource constraints.

3. Stochastic activity duration (and cost).