A behavioral multi-agent model for road traffic simulation

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Abstract

Multi-agent systems allow the simulation of complex phenomena that cannot easily be described analytically. Multi-agent approaches are often based on coordinating agents whose actions and interactions are related to the emergence of the phenomenon to be simulated. In this article, we focus on road traffic simulation, specifically the design of a road traffic simulation tool able to deal realistically with road junctions. We propose a multi-agent behavioral model based on (i) the opportunistic individual behaviors that describe the norm violation and (ii) the anticipatory individual abilities of simulated drivers that allow critical situations to be detected. Our proposition has been validated for different traffic scenarios. Specifically, we simulated the traffic in a real intersection and then compared the simulated traffic flow with the real flow to highlight the relevance of our approach.

keywords : multi-agent coordination, anticipatory model, norm violation, traffic simulation

1 Introduction

Simulation tools are very complex softwares whose development can be facilitated by considering a multi-agent approach. A simulation process consists of reproducing the dynamic behavior of a real phenomenona using models, simplified abstract mental constructions of reality that allow the phenomenona to be simulated. In most simulation tools, the dynamic behaviors of the simulated phenomenona are expressed using mathematical techniques, such as differential equations, for instance. Such mathematical approaches take advantage of the universality of the mathematical language, but still usually reach their limits when large-scale, complex or highly dynamic phenomena must be simulated.

The multi-agent paradigm (Genesereth (1991); Sycara (1998); Ferber (1999); Weiss (2000)) offers one response to the limitations of traditional approaches to simulation. Multi-agent systems (MAS) can help to simulate complex and dynamic phenomena - autonomous vehicle steering (Sukthankar et al. (1998)), pedestrians in an urban environment (Grislin-Le Strugeon et al. (2004)) - phenomena that are difficult to express using classic mathematical formalisms. Multi-agent simulation is based on the interaction of agents whose actions and interactions are related to the emergence of the phenomena to be simulated.

In this paper, we consider the issue of traffic simulation by reproducing real observed traffic phenomena. Road traffic can be defined as the set of complex phenomena resulting from the moves of road users on a limited-capacity road network. The "traffic system" includes both the road infrastructure designed to respond to a collective optimum and the increasing demands of users who are all trying to obtain an individual optimum.The complexity of the issues involved means that traffic simulation tools are used for many applications: forecasting and controlling traffic systems (Hall (1997)), optimizing traffic flows (Bazzan (2005)), as well as studying new road profiles (Espié et al. (2002)), to name but a few.

One of the hardest problems in the development of road traffic simulation tools is the case of intersection. The common approach, used in most traffic simulation tools, involves a drastic simplification of the problem. A "solver", which has to manage traffic inside the intersection, lets vehicles enter only when their trajectories are not in conflict. Such solutions are sometimes sufficient, but are not acceptable when the aim of the simulation is to mimic the actual behavior of real drivers. The accuracy of such simulations is all the more important because driver behavior in intersections influences the overall flow on the road network.

This article deals with a multi-agent model that is able to realistically simulate the traffic inside an intersection. The next section gives a review of existing approaches to road traffic simulation, specifically traffic simulation at road junctions, including both traditional and multi-agent approaches. The third section describes our multi-agent model for simulating traffic at road junctions. This model is a part of ArchiSim, the behavioral traffic simulation model developed by the French National Institute for Transport and Safety Research (INRETS). The fourth section describes our experiments based on specific traffic scenarios, and the results generated by our model are compared to the traffic flow measurements taken at real intersections. The last section presents our conclusions and offers our suggestions for future research.

2 A review of existing traffic simulation tools

All traffic simulation tools do not provide the same level of detail. These tools are currently classified according to their level of granularity: macroscopic, mesoscopic and microscopic (Williams (1997)). In the following sections, we will briefly discuss each of the major approaches.

2.1 Centralized approaches for traffic simulation

The first traffic flow models were mathematical. Such a model allows, for instance, a highway traffic situation to be modeled using car-following laws, which are, in fact, differential equations that are obtained empirically through regression using data collected at currently operating road sections (Lieberman and Rathi (1997)).

Even now, most of the microscopic simulations[1] use the car-following law to model in-line driving, while the specific case of the intersection is managed using centralized scheduling techniques. In these applications, each vehicle approaching the intersection is placed in a virtual queue - one for each branch of the intersection. The head of each queue moves according to a centralized process based on the principle of "gap acceptance". This search process looks for spaces through which vehicles can cross or in which the vehicles can be inserted. For a driver, "gap acceptance" corresponds to an acceptable inter-vehicular time (i.e., the time between two successive cars in conflicting flows) during which a maneuver can be performed. This "gap acceptance" principle can be generalized for a succession of intersections (i.e., an urban axis) by introducing simple tricks to manage the side effects between consecutive intersections. For instance, Vissim (Vissim (2005)) has a "yellow box" parameter, which allows users to define a minimal speed that the vehicles inside the intersection must respect so that other vehicles can enter the intersection.

[1]Microscopic simulations consider each vehicle individually, while macroscopic simulations focus on vehicle streams.

As this brief overview of existing traffic simulation tools suggests, the centralized scheduling approach is the most often used. A centralized scheduler makes decisions for each vehicle, similar to a policeman who lets the vehicles enter the intersection only when their trajectories are not in conflict. However, using car-following laws and scheduling techniques has several limitations. First, such approaches are not and cannot be generic. For instance, because car-following laws are obtained from measurements taken on existing roads, the resulting equations are linked to the characteristics of the actual road section (length, number of lanes, road markings) and thus only allow replays of the particular traffic situation observed on this road section. Second, although a scheduler may be sufficient for studying the traffic flow input and output in an intersection, schedulers are inappropriate for studying the phenomena inside the intersection. Indeed, the simulated behaviors of individual drivers produced by schedulers are not always realistic, and thus many traffic phenomena can not be simulated (e.g., traffic backup between intersections, traffic signals violation and traffic congestion inside the intersection). Using a scheduler centralizes the decision-making of all the drivers in the intersection, and consequently, the simulated interaction is far different than the real interactions of human drivers. One way to get around the limitations of the centralized scheduling approach is to use a behavioral approach to decentralize traffic simulation.

2.2 Behavioral approaches

Unlike the mathematical approach described above, the behavioral approach considers traffic as an emerging phenomena resulting from actions and interactions of the various traffic system actors (e.g., car drivers, pedestrians, road operators). The behavioral approach aims to accurately model and reproduce the behaviors and interactions of the simulated entities in order to obtain realistic traffic phenomena. The realism of the emergent traffic also depends on the distribution of behaviors among the set of simulated entities (i.e., heterogeneity of the individual practices).

2.2.1 Cellular automata approaches

Some models using cellular automata have been introduced to solve the problems of traffic simulation at intersections. For instance, Ruskin and Wang (2002) have used a deterministic cellular automata that discretizes the intersection on a grid whose cells can either contain a vehicle or be empty. The vehicles movements take advantage of this formalism and vehicle behaviour is considered to be a transition function of the automata. Nevertheless, using a grid to represent the environment is too limited to correctly render the dynamics of traffic situations at intersections. In fact, discretizing the driving area leads to discretizing vehicle speed, thus allowing only three speeds: $0km/h$, $25km/h$ and $50km/h$.

2.2.2 Robotics-inspired approaches

Modeling the driving task has been also examined from a robotic point of view. Some computing-based models have been proposed by robotics specialists, with the objective of creating a robot capable of evolving with the road traffic, first in simulation and then in reality. For instance, Reece and Shafer (1993) proposed a case-based reasoning model for intersection situations. Though the resulting model behavior is appropriate for a robotic context - the robot is able to drive safely without causing any collisions - it is not really appropriate in the traffic simulation context since the model is not able to recreate all the individuality of real driver behaviour.

2.2.3 Multi-agent systems approaches

In the traffic simulation literature, two models seems us interesting, providing a real alternative to classic traffic simulation models.

The first one, proposed by Trannois et al. (1998), is an adaptation of the well-known blackboard system for planning the actions of agents evolving in an urban environment. The originality of this approach lies in the model of the environment and the distribution of the knowledge about the driving task. This knowledge is not integrated into each agent's strategy but is instead distributed among the entities on the road infrastructure. This kind of distribution means that, for example, a yield sign will bring the knowledge needed to cross the intersection to the attention of the different agents. All knowledge received by an agent is arranged in order, according to its relevance in the traffic situation. The inconvenience of this approach is that driver behaviour is directly connected to a the road infrastructure. Consequently, with this model, it is very difficult to obtain agents that are autonomous in terms of the Highway Code (norm autonomy).

The second one was proposed by Paruchuri et al. (2002). They introduced a model in which each simulated driver is autonomous, making its own decision using fine-tuning parameters (e.g., autonomous braking power, maximum braking power, autonomous acceleration, maximum acceleration, minimum inter-vehicle gap, overtake margin). These authors claim that their model allows realistic driver behaviour to be simulated. However, the autonomous adaptivity of the agents is limited since some traffic situations (e.g., gridlock situations) must be supervised and controlled by an external centralized process.

2.3 Towards a more realistic behavioral model

The multi-agent model is, in our opinion, the most relevant computing model for microscopic traffic simulation, allowing more accurate and realistic drivers' behaviors to be simulated in a dynamic environment. We think that studies of driving psychology can help to build and enrich multi-agent models for traffic simulation. Indeed, these studies provide a better understanding of the reasons underlying real driver decision-making, by revealing specific facets and charateristics of driver behavior. For instance, they show that drivers tend to:

- maintain their preferences: desired speed, road position;
- act differently depending on which country they live in (Summala (2005));
- be opportunistic, selectively respecting the Highway Code depending on the context (Björklund and Åberg (2005)); and
- anticipate traffic situations.

In the next section, we present our multi-agent behavioral model for microscopic traffic simulations. This model is based on a coordination mechanism and includes both opportunistic and anticipatory behaviors.

3 A behavioral traffic simulation model

Our multi-agent model for simulating the traffic in intersections is a part of the ArchiSim project.

3.1 ArchiSim : behavioral traffic simulation tool

ArchiSim is a behavioral traffic simulation tool. Developped in 1992 by the French National Institute for Transport and Safety Research (INRETS) (Espié (1995)), this tool is still evolving. The originality of ArchiSim, compared to the other traffic simulation tools, lies in its ability to make the driving simulator[2] a part of the traffic simulation. In other words, ArchiSim is able to generate a realistic traffic environment for a human driver using a simulator: the simulated drivers interact with the driving simulator as they would with any of the other vehicles in the simulation.

The ArchiSim architecture allows the simulation to be distributed over a set of machines connected to each other via an Ethernet network, meaning that the actors in the simulation can act on different machines. The coherence of the whole is ensured by a server that contains all the data related to the simulation process: the road network description (e.g., road geometry, infrastructure and traffic signs) and vehicle information (e.g., position in the network, current speed and acceleration). This server centralizes the visual information given to the various agents : each entity in the simulation, be it virtual or real, is connected to the server and receives a set of information about its close environment at each time step during the simulation. After making its decision, each simulated driver updates its own information on the server.

ArchiSim has been validated for Highway situations (Champion et al. (2002), El Hadouaj et al. (2004)) and preliminary research has been conducted to extend the ArchiSim traffic model to road traffic in intersections (Champion et al. (2003)). The next section presents the recent advances in the traffic model that allows ArchiSim to deal with complex traffic phenomena.

[2]This simulator is a device composed of a cabin and a projection system that immerses a human driver in a virtual traffic situation.

3.2 Traffic simulation at road junctions: a multi-agent coordination issue

The case of road junctions is considered as a multi-agent coordination issue. When crossing an intersection, real drivers solve any conflict that arises with other vehicles. In simulation, this conflict-resolution driving task can be expressed as a coordination problem because all agents approaching an intersection have to coordinate their actions in order to avoid collisions. The problem is difficult because the driving task appears to differ from one country to another (Saad (1992), Björklund and Åberg (2005), Summala (2005)). In fact, in southern Europe and Asia, driving is highly competitive, especially for Latin drivers. In northern Europe, however, it is less competitive and more cooperative.

The coordination mechanism used in ArchiSim is designed to be flexible and generic in order to take all the features described above into account and to reproduce a large variety of driver behaviors. Based on the model of conflicts inside the intersection, this mechanism breaks down the complex interactions between agents into elementary situations involving two agents.

A real driver perceives a complex road intersection as a succession of elementary T-type or X-type intersections. A roundabout, for example, can be seen as a succession of T-type intersections, while a 4-corner stop is a simple example of an X-type intersection. The interactions between drivers in an elementary intersection are regulated by the priority relations defined in the Highway Code and individual practices depending on countries. Such a priority relationship can be expressed by the predicate *prio*. For example, let us consider two agents x and y approaching an intersection. The four elementary situations presented in the figure 1 are possible: 1) x and y are on the same road and no one turns, meaning that no conflict exists between x and y; 2) x arrives to the right of y and in absence of any signals (e.g., yield signs, stop signs or traffic lights), x has priority; 3) y arrives to the right of x and consequently the priority; 4) x and y are on the same road and both turn left, which, according to the Highway Code, means they both have the priority.

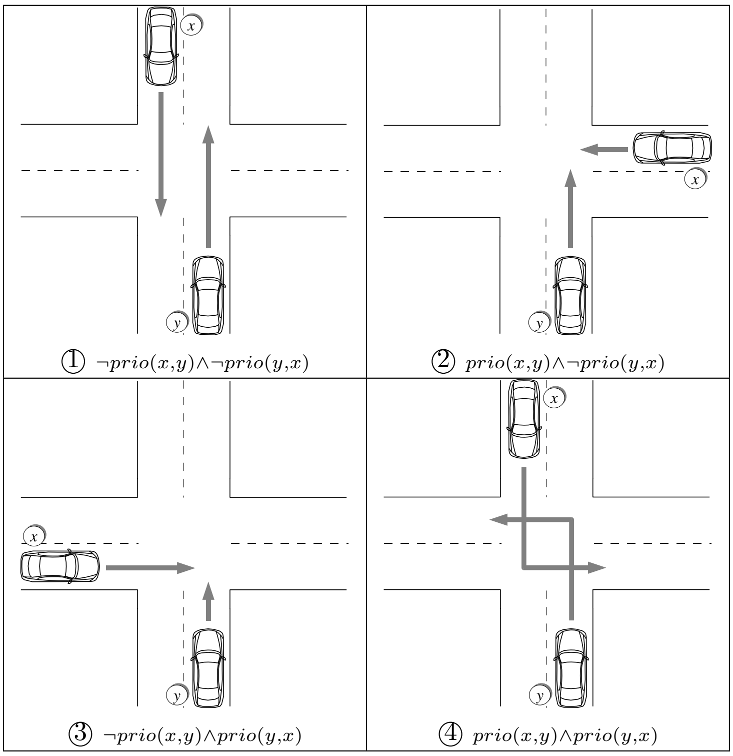

Figure 1: Elementary situations in an intersection (Champion et al. (2003))

The coordination mechanism (Mandiau et al. (2008)) has the following dynamic behavior. An agent approaching an intersection searches for all the vehicles with which it is potentially in conflict and assesses its priority relationship with each of those vehicles. Each priority relationship is used as a local rule that indicates whether the agent must speed up or slow down. When multiple priority relationships are involved in the current situation, the agent chooses the behavior that will lead to the lower speed. At this point in the ArchiSim development, the coordination algorithm only manages the longitudinal acceleration, meaning that the agent can choose between two actions: *Stop* or *Go*. Each agent decision depends on how the agent interprets the priority relationships. In the following section, we apply this coordination scheme to the behavioral model of the agents.

3.3 The opportunistic part of the model

The coordination mechanism described above is primarily based on the perceptions of the priorities relationships defined in the Highway Code. But, as we mentioned in the first section, driving psychology studies have shown that the Highway Code are not always respected and applied. In real life, drivers move their cars opportunistically: depending on the context and the traffic situation they may choose to ignore the rules of the Highway Code in order to temporarily satisfy their own desires. The importance of this particular facet of driver behavior has been highlighted by recent works that attempt to predict intentional driver violations (Chaali-Djelassi et al. (2006)).

To allow an agent to violate the rules of Highway Code and thus depict opportunistic behavior, we chose to modify the agents' priority recognition (Doniec et al. (2006b)). In the following discussion, the priority relationship described in the Highway Code is referred to with the predicate *prioCode*. If the driver's recognition of this priority relationship is based only on the traffic signals that an agent perceives and its knowledge of the Highway Code, the resulting behavior (after coordination) should be normative. Indeed, the priority relationship *prio* used by the coordination mechanism is directly mapped from the predicate *prioCode*: $prioCode(\alpha, \beta) \rightarrow prio(\alpha, \beta)$.

Such a simple priority recognition can be enriched by considering other contextual information about the agent's driving situation. The following information, based on individual practices that are frequently observed in reality, can have a real impact on priority recognition (Björklund and Åberg (2005)):

- **speed differences:** drivers who have the priority at an intersection (from the point of view of the Highway Code) tend to slow and cede that priority if they observe another vehicle approaching the intersection at a high speed. On the other hand, the drivers of high-speed vehicles continue to violate the Highway Code since they feel they have enough time to cross the intersection before the lower-speed vehicle reaches it.

- **impatience levels:** drivers who have been at a stop for a long time tend to become impatient and are likely to feel that they have priority over other vehicles, even when the Highway Code would say otherwise.

- **vehicle positions:** drivers who, due to high traffic density, must cross an intersection in two movements tend to feel uncomfortable waiting in the middle of the intersection. Their perception of this position as dangerous often moves the drivers to decide that they have the priority, even when this decision would be contrary to the Highway Code.

Since real drivers appear to exercise self-control when violating the norms, we propose to establish an auto-controlled norm violation, considering the following rules, expressed as:

$$\begin{aligned} &\boxed{\textit{normative behavior}} \wedge \boxed{\textit{violation precondition}} \\ \wedge &\boxed{\textit{violation postcondition}} \rightarrow \boxed{\neg\textit{normative behavior}} \end{aligned}$$

This rule can be broken down into four parts:

- The part dealing with *normative behavior* expresses the priority relationship between the agent's vehicle and another vehicle from the point of view of the Highway Code.

- The part dealing with the *violation precondition* expresses the fact that the agent is planning to violate the Highway Code and is acting in a non-normative way.

- The part dealing with *violation postcondition* expresses the condition that must be fulfilled for the agent to be able to act in a non-normative way.

- The part dealing with the *non-normative behavior* expresses the priority relationship that the agent has to perceive in order to act in a non-normative way.

In the following paragraphs, we describe some of the rules that have been implemented in ArchiSim. Consider the traffic situation shown in figure 2. An agent x is at a stop sign and has to determine its priority relationships with the vehicles y and y'. According to the Highway Code:

- y and y' are driving on the main axis, and x is sitting at a stop sign: both y and y' have priority over x.
- x must come to a full and complete stop of at least three seconds.
- x should not enter the intersection if it is not sure that it will be able to exit in one movement.

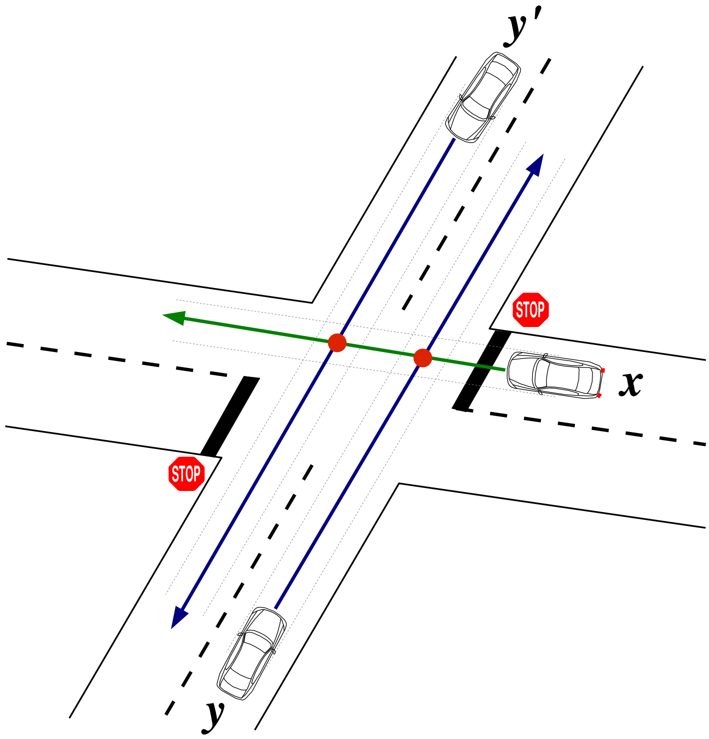

Figure 2: An intersection with a Stop sign and the agents' coordination choices

To describe each of the four parts of the rules, the following predicates must be introduced:

- *impatience* which indicates the agent's level of impatience (this level is activated when the agent is stopped for one interval longer than a certain time threshold, which differs from one agent to another).
- *restarting* which specifies whether or not the agent is in an acceleration phase after a period at a complete stop (this predicate remains true until the agent reaches a certain speed threshold).

Moreover, a function *distC*, which expresses the distance the two agents must cover before coming into conflict, is also introduced.

When the traffic inside the intersection is relatively dense, total compliance with the Highway Code rules leads to extremely long waiting periods at the Stop sign. In real life, a driver usually chooses not to respect the Code's rules. For example, when the vehicle y is stopped or proceeds at a very low speed, agent x may decide that it has the priority (*non-normative behavior*) if it is impatient (*violation precondition*) and is closer to the point of conflict than y (*violation postcondition*).

$$\begin{aligned} &\boxed{\neg prioCode(x,y) \wedge prioCode(y,x)} \wedge \\ \boxed{impatience(x)} \wedge &\boxed{(distC(x,y) < distC(y,x))} \\ \longrightarrow &\boxed{prio(x,y) \wedge \neg prio(y,x)} \end{aligned} \tag{1}$$

If vehicle y is proceeding at a significant speed, it really makes no sense to consider the distance to the point of conflict expressed in the precondition part of the rule; it is better to compare the time to the point of conflict. When agent x is stopped at the Stop sign, its time to conflict can be calculated with a prevision γ_x^* of its future acceleration if it should decide to move forward. The time to conflict is therefore calculated using the kinetic formula: $\sqrt{\frac{2\, distC(x,y)}{\gamma_x^*}}$. Vehicle y's time to conflict can be obtained simply by considering its instantaneous speed V_y without taking its acceleration into account. The following rule defines the situation: if vehicle x has no priority over vehicle y according to the Highway Code (*normative behavior*), but if x is impatient (*precondition*) and has enough time to cross the intersection safely (*postcondition*), then x may consider that it has priority.

$$\begin{array}{r} \boxed{\neg prioCode(x,y) \wedge prioCode(y,x)} \wedge \\ \boxed{impatience(x)} \wedge \boxed{\sqrt{\frac{2\, distC(x,y)}{\gamma^*}} < \frac{distC(y,x)}{V_y}} \\ \longrightarrow \boxed{prio(x,y) \wedge \neg prio(y,x)} \end{array} \tag{2}$$

The precondition part of these rules describes a particular state of the agent. Other rules can be defined to consider the fact that the agent is beginning to move again and going to enter the intersection. At this point in time, even if x detects a vehicle y that has priority, x must continue to move. To act safely, x must check to be sure that it has enough time to cross the intersection before y reaches the conflict point (*postcondition*).

$$\begin{array}{r} \boxed{\neg prioCode(x,y) \wedge prioCode(y,x)} \wedge \\ \boxed{restarting(x)} \wedge \boxed{\sqrt{\frac{2distC(x,y)}{\gamma^*}} < \frac{distC(y,x)}{V_y}} \\ \longrightarrow \boxed{prio(x,y) \wedge \neg prio(y,x)} \end{array} \tag{3}$$

Now, let us consider the case of a vehicle approaching the intersection at a high speed. This case can also be illustrated by figure 2 if the stop sign is replaced by a yield sign. If the speed of x is sufficiently high compared to the speed of y, x may consider that it has priority over y. This case is characterized by the following rule:

$$\begin{array}{r} \boxed{prioCode(x,y) \wedge \neg prioCode(y,x)} \wedge \\ \boxed{\frac{distC(y,x)}{V_y} < \frac{distC(x,y)}{V_x}} \wedge \\ \boxed{distC(y,x) < -\frac{{V_y}^2}{2\gamma_y}} \longrightarrow \boxed{\neg prio(x,y) \wedge prio(y,x)} \end{array} \tag{4}$$

In this rule, the post condition $distC(y,x) < -\frac{{V_y}^2}{2\gamma_y}$ expresses the possibility that y will able to slow down with γ_y as the deceleration and stop before the point of conflict, with the kinetic formula $-\frac{{V_y}^2}{2\gamma_y}$ expressing the stopping distance.

The behavioral model for a simulated driver is completed by other rules (Doniec et al. (2006b)), which are not described in this paper. This model realistically defines an agent's interactions with the other agent vehicles in an intersection (*non-normative behavior* part of the rules) without risk of collision (*postcondition* part of the rules). But, this is not enough to ensure the social order of the traffic flow. The Highway Code was introduced not only to prevent collisions between cars, but also to avoid gridlock situations. To accomplish this second objective, we propose to enrich the *postcondition* part of our rules with an *a priori* evaluation of the risk of gridlock.

3.4 The anticipatory part of the model

3.4.1 Context and model

In the previous section, the introduction of opportunistic behavior requires allowing some of the rules of the Highway Code to be violated. The Highway Code rules were introduced not only to avoid collisions, but also to ensure a smooth and continual flow of traffic. For instance, a Highway Code rule says that drivers must not enter an intersection if they are not sure to be able to exit the intersection in a single movement. Since this rule is rarely applied by real drivers, this rule was not implemented in our model. Consequently, in high density traffic simulations, agents tend to gather in the center of the intersection, thus creating gridlock (figure 3).

Figure 3: Gridlock in the center of an intersection during a traffic simulation

In some experiments, we have noticed that the number of simulated gridlocks remains high compared to real traffic situations[3]. In real life, even though drivers behave individually, the number of gridlocks remains low since drivers anticipate (Saad (1992)). To keep the simulation realistic, we decided to complete our model with an anticipatory layer, thus providing agents with anticipatory abilities. To formalize this part of our model, we chose a particular type of anticipation: preventive anticipation.

Preventive anticipation was first described by Rosen (Rosen (1974)). Consider a system S and its model M. To make predictions, M is simulated faster than S. To modify the behavior of S according to these predictions, Rosen proposed partioning *"the state space of S (and hence of M) [...] into regions corresponding to desirable and undesirable states. As long as the trajectory in M remains in a desirable region, no action is taken by M"*. In other words, preventive anticipation is viewed as the process of trying to keep the system in a desirable state.

To model and implement preventive anticipation, we use a constraint processing approach. A constraint network is a set of variables with associated domains and a set of constraints. *"Variables are objects or items that can take a variety of values. The set of possible values for a given variable is called domain. [...] Constraints are rules that impose a limitation on the values that a variable may be assigned." (Dechter (2003)).* Our model for preventive anticipation is based on each agent constructing a mental representation of its environment, specifically of the different relationships and

[3]In the section 4.1.2, we show that the number of gridlock situations during initial simulations could reach 35 for a one-hour simulation.

interactions existing between the agents in the environment (Doniec et al. (2006a)). The structure of this mental representation includes the three components of a constraint network: the variables, the domains and the constraints.

To describe our approach, we use the following notations:

- $\mathcal{A}$ the set of agents (simulated drivers); and
- $\mathcal{E}$ the environment and $I_{\mathcal{E}}^{a_i}$ the set of all information perceived by an agent $a_i \in \mathcal{A}$.

A mental representation M_{a_i} of an agent a_i is a triplet (A, R, D) in which:

- $A = \{a_1, a_2, ..., a_n\}$ is the subset of all agents perceived by a_i;
- $R = \{r_1, r_2, ..., r_k\}$, where each r_i is a binary relation expressing an interaction between two agents of A; and
- $D = \{dom(a_1), dom(a_2), ..., dom(a_n)\}$ defines the domain of each agent in A.

The nature of the domain can vary from one application to another. For example, a domain can be spatial or temporal.

Using the mental representation that it has constructed, each agent tries to predict the future state of the system, or, in other words, tries to determine how its actions will affect the system's evolution. In our anticipatory model, we consider that each action has direct and indirect effects. A direct effect of an action is modeled by adding or deleting a relation in the set R. Indirect effects are calculated by using propagation techniques over the direct effects.

Our approach can be summarized by the algorithm in figure 4. Our anticipation algorithm scans a list of actions LA and eliminates those actions that induce an undesirable state contained in the list LUS. Undesirable states are expressed as a list of particular domains, which is a parameter of the algorithm. The first step of the *anticipate* procedure begins by making a first propagation over M_{a_i}, using an external propagation function. Classic algorithms such as AC-3 (Mackworth (1977)), can be used and adapted to the application. The direct effect of each action in the list LA is then evaluated, using an external function that is specific to the application. In our traffic simulation example, this external function uses a kinetic calculation based on the current position and vehicle speed, to determine the direct effects. The direct effects of an action are then used to update the agent's mental representation M_{a_i}. Based on this update, a search for undesirable states is performed; this operation verifies that the intersection between the set D and LUS is not empty. If an undesirable state is found, the action that caused that it is removed from the list LA.

procedure *anticipate*
  in: Actions LA,UndesiredStates LUS,
  $M_{a_i}(A, R, D)$, $I_{\mathcal{E}}^{a_i}$
  out: Actions LA

begin
$propagate(M_{a_i})$
$M' \leftarrow M_{a_i}$
for each $action \in LA$ **do**
    $DE \leftarrow computeDirectEffects(a, I_{\mathcal{E}}^{a_i})$
    $update(R, DE)$
    $propagate(M_{a_i})$
    if $D \cap LUS \neq \{\}$ **then**
        $LA \leftarrow LA - \{action\}$
    $M_{a_i} \leftarrow M'$
end

Figure 4: The *anticipate* procedure

3.4.2 Propagation relations

To use our anticipatory model to simulate road traffic, we also had to define the three components of the mental representation M_{a_i} of an anticipatory agent a_i. The set A is built using the agents present in the intersection and perceived by a_i.

The set R is composed of (i) the previously introduced priority relationships (*prio* predicates) and (ii) three types of "blocking" relationships:

- $bph_{a_i}(a_x, a_y)$, meaning "a_x is physically blocked by a_y from the point of view of agent a_i";
- $bpha_{a_i}(a_x, a_y)$, meaning "a_i observes that a_x will be physically blocked by a_y"; and
- $bpr_{a_i}(a_x, a_y)$, meaning "a_y has priority over a_x from the point of view of the agent a_i".

The domain associated with each agent in A is temporal and expresses the next time step of the simulation. The domains are finite and are expressed as a set of intervals. The upper bound of the domain is a specific parameter for each agent, expressing the time distance that the agent is able to anticipate. Please note that the domains depend on the agent a_i. For reasons of simplicity, the notation has been shortened from $dom_{a_i}(a_z)$ to $dom(a_z)$. At the current time step t, if a value x is not present in the domain of a perceived vehicle, this vehicle will be blocked at $t + x$. For instance:

- $dom(a_z) = \{[1, 20[\}$ means that "a_z can act and move during the interval from $t + 1$ to $t + 20$"; and
- $dom(a_z) = [1, 4] \cup [8, 10]$ means that "a_z will be blocked between $t + 5$ and $t + 7$".

Since the relations all have a specific semantic and are used with temporal domains, we have defined specific propagators for each of them. These propagators use interval processing techniques.

The *bph* relation can be expressed as the difference between two time intervals corresponding to the time the blocked vehicle needs to restart:

$$bph_{a_i}(a_x, a_y) \equiv_{def} \Big(dom(a_x) = \Big\{[t_{a_y} + Tr(a_x), t'_{a_y} + Tr(a_x)]\Big\}\Big) \quad (5)$$

where $Tr(a_x)$[4] is the time needed by the vehicle a_x to start again, and t_{a_y} and t'_{a_y} are, respectively, the upper and lower bounds of $dom(a_y)$. The propagation of *bph* is illustrated in figure 5.

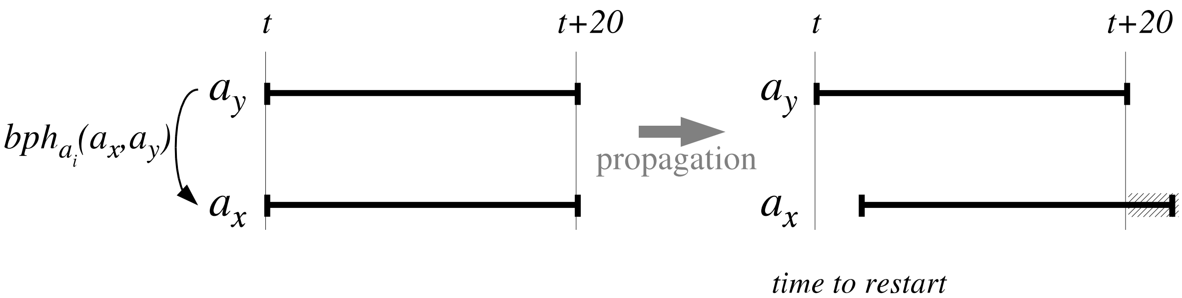

Figure 5: Propagation of the *bph* relation

The *bpha* relation can be defined as the fusion of the time interval needed for the blocking action to take effect and the domain of the blocking vehicle.

$$bpha_{a_i}(a_x, a_y) \equiv_{def} \Big(dom(a_x) = [t_{a_x}, t_{a_x} + Ttc(a_x, a_y)] \cup [t_{a_y} + Tr(a_x), t'_{a_y} + Tr(a_x)]\Big) \quad (6)$$

[4] The French acronym for "restarting time".

where $Ttc(a_x, a_y)$[5] is the time a_x needs to reach a_y and t_{a_x} is the lower bound of $dom(a_x)$. Figure 6 represents the propagator of the relation *bpha*.

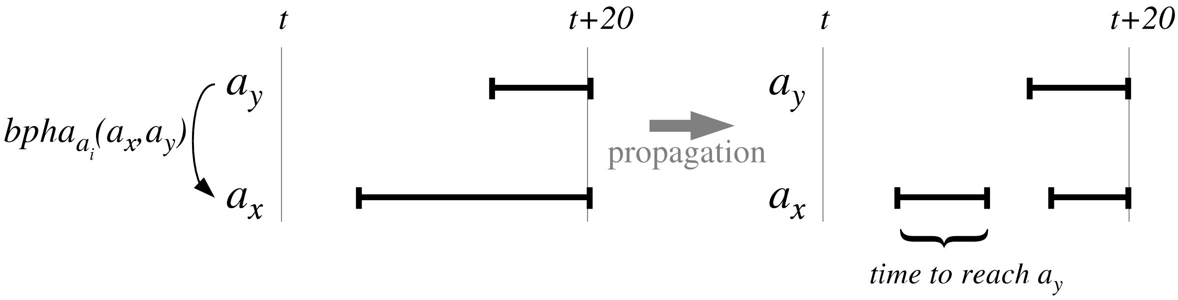

Figure 6: Propagation of the *bpha* relation

The *bpr* relation can be described as the difference between the time interval needed by the blocking vehicle to get past the conflict point and its own domain.

$$bpr_{a_i}(a_x, a_y) \equiv_{def}$$
$$\Big(dom(a_x) = \Big\{[t_{a_x}, t'_{a_x}] - [t_{a_y}, t_{a_y} + Tpc(a_y, a_x)]\Big\}\Big) \quad (7)$$

where $Tpc(a_y, a_x)$[6] is the time interval during which a_y has not yet gone past the conflict point and is still on the trajectory of a_x. This case is illustrated in figure 7.

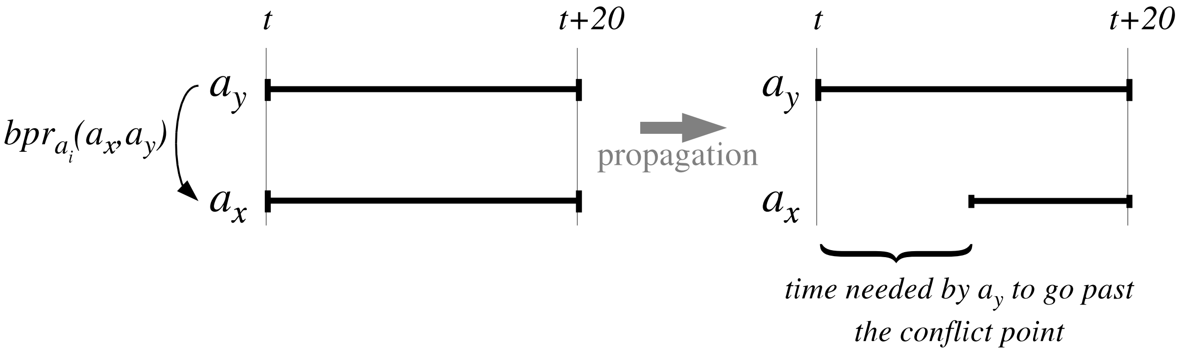

Figure 7: Propagation of the *bpr* relation

3.5 'Genericity' of the model

The model described above is deterministic and can produce identical simulations for two consecutive runs. Nonetheless, each agent can be differentiated individually and can be fine-tuned using personnal parameters that influence the agent's decision.

Thus, calculating the rules presented in section 3.3 is done with kinetic calculus based on internal agent parameters. For instance, the calculation of *distC* takes into account a security margin that is different from an agent to another. This parameter is very useful for fine-tuning an agent's behavior. If an agent is intended to simulate a Latin driver, it would probably be best, given the researh on such drivers' behavior, to set a low security margin. On the contrary, when simulating a Scandinavian driver, the agent's security margin should certainly be high.

The anticipation mechanism presented in section 3.4 also uses individual parameters that differs from one agent to another. For example, the upper bound used by agents to compute the temporal

[5] The French acronym for "time to conflict".
[6] The French acronym for "time to past conflict".

domains is an individual parameter that make it possible for agents to have different anticipatory capacities with respect to predicting future events). Moreover, all temporal calculations (e.g., time to conflict, time to past conflict) are also based on kinetic calculus using internal agent parameters.

The set of all agent parameters gives the model a certain generic quality since it is possible to simulate a large variety of drivers in a large variety of driving situations. The next section will present the experiments conducted to evaluate our model for simulating road traffic at an intersection.

4 Experiments and validation

The coordination mechanism proposed by Champion et al. (2003) aims to avoid collisions between simulated vehicles, and a comparison of the simulated traffic flows produced using this mechanism and the real traffic flows in the main branches of certains intersections shows that the results are accurate. However, this basic behavior model is unrealistic for some traffic situations, such as unjustified evolutions of simulated vehicle speeds when approaching an intersection, the appearance of gridlock situations at the center of the intersection, and the insertion practices of vehicles crossing a vehicle stream. The results of the experiments presented in this section shows that the behavioral model described above greatly improves the traffic coordination.

First, a variety of traffic scenarios were used to evaluate the individual behavior of the simulated drivers in different traffic situations, including some extreme traffic situations, such as gridlock. Then, the results were verified to insure that the individual behaviors reflected reality and revealed the actual traffic situation. We validated our approach by simulating the road traffic in a real intersection over a one-hour period.

4.1 The evaluation of individual behaviors in the different traffic scenario

4.1.1 Reducing Oscillation

The first experiment was designed to illustrate the reduction of the oscillation phenomena in terms of vehicle speed. Oscillation phenomena are well-known problems in multi-agent systems, particularly in dynamic simulations. They generally involve decision-making oscillations: the agents hesitate between two actions and alternate between one and the other. Oscillation problems are a severe drawback in a multi-agent reactive approach. For instance, Koren and Borenstein (1991) have shown that oscillation is one of the main weaknesses of the potential fields used for autonomous navigation in robots. Some oscillation phenomena can be emergent: they appear in particular situations, in which the agents successively enact a certain combination of actions, repeating themselves cyclically.

In the scenario considered here, a vehicle approaching an intersection does not have priority over the other vehicles. To avoid accidents and to insure that the intersection is crossed safely, the vehicle has to slow down. Figure 8 shows the first vehicle speed curve produced with the basic algorithm (Champion et al. (2003)). Two oscillation phases are noticeable in the curve. The first one is between the time steps 50 and 125 with a speed varying around a median value of 8 km/h. The second phase is between steps 200 and 275 with the speed varying between 1 and 5 km/h.

Introducing an opportunistic behavior reduced the oscillation phenomena due to oscillations in the priority perceptions. The second curve in figure 8 presents the deceleration of an agent whose behavior is opportunistic in conditions identical to those described just above. As the figure shows, a stable speed was maintained in both time periods (i.e., between steps 50 and 125, and between steps 200 and 225) and the deceleration curve is smoother and less irregular.

4.1.2 Reducing gridlock

The second experiment used anticipatory behavior of simulated drivers to reduce gridlock. To illustrate this point, we considered a X-type intersection with several traffic flow variations in each simulation. For each one, gridlock situations were counted via a supervision process that, though part of the simulator, is totally independant of the agents' decision-making process. At each time step, this supervisor checks to see whether or not gridlock is produced. If gridlock is detected, the simulation is suspended, the intersection is completely cleared, and then the simulation takes up where it left off.

Figure 9 shows how anticipation reduces the number of gridlock situations during the simulation of a non-signaled intersection. For a one-hour simulation using non-anticipatory agents, the number of

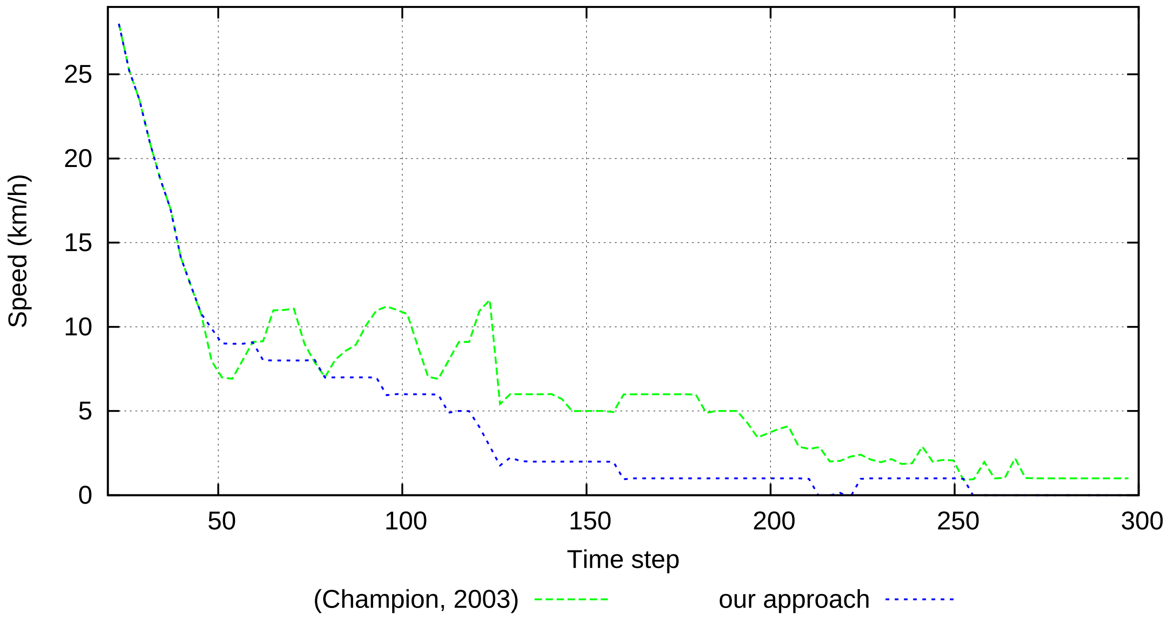

Figure 8: Deceleration of an agent when approaching in intersection

gridlock situations varies from 0 to 35, depending on the traffic flow. When agents have anticipatory abilities, the number of gridlock situations remains near zero almost all the time for simulated flows of up to 800 vehicles/hour per axis.

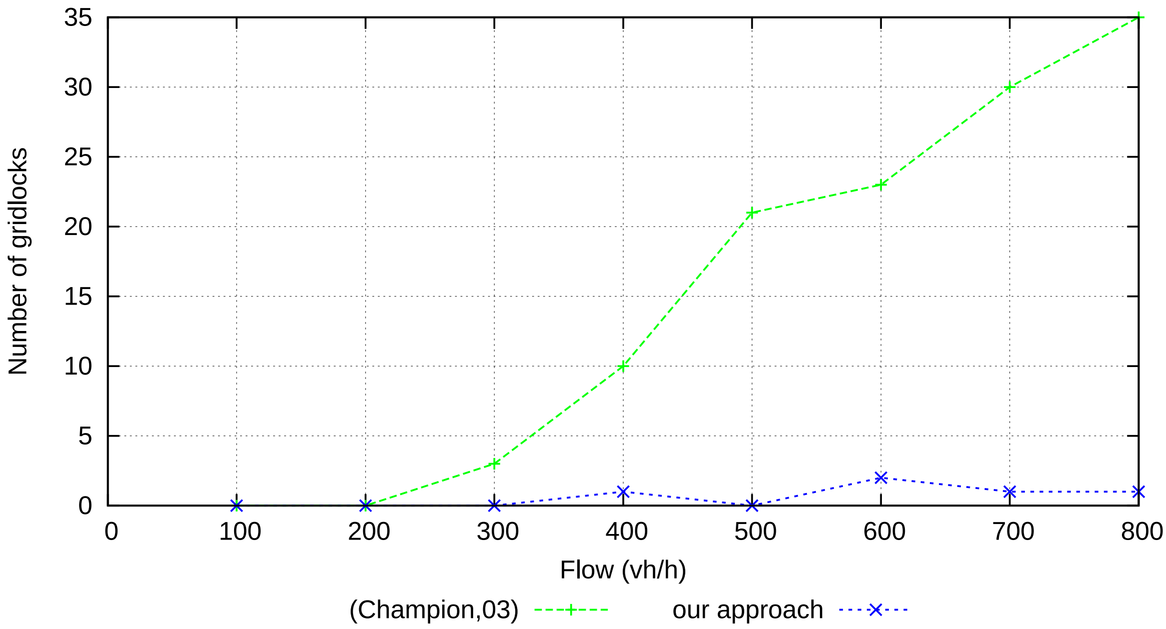

Figure 9: Variation of the number of gridlock situation versus the traffic flow

Other gridlock situations have already been tested successfully: traffic simulation with a series of double left turns, traffic backup between intersections, and large vehicle (e.g., truck, bus) movements inside an intersection (Doniec et al. (2005), Doniec et al. (2006a)).

Figures 10 and 11 illustrate the case of large vehicles approaching an intersection, given two possible behaviors: one without anticipation and one with anticipation. In figure 10, the agents have no anticipatory abilities. At step 2154 (figure 10.a), the buses are approaching the intersection and both want to turn left. At step 2175 (figure 10.b), bus 23 decides to enter the intersection and one second later (step 2189, figure 10.c) begins its turn. However, bus 24 also decides to enter the intersection and its length combined with that of bus 23 creates gridlock. The same situation involving passenger vehicles rather than buses would not have created gridlock since the vehicles would have be able drive around one another.

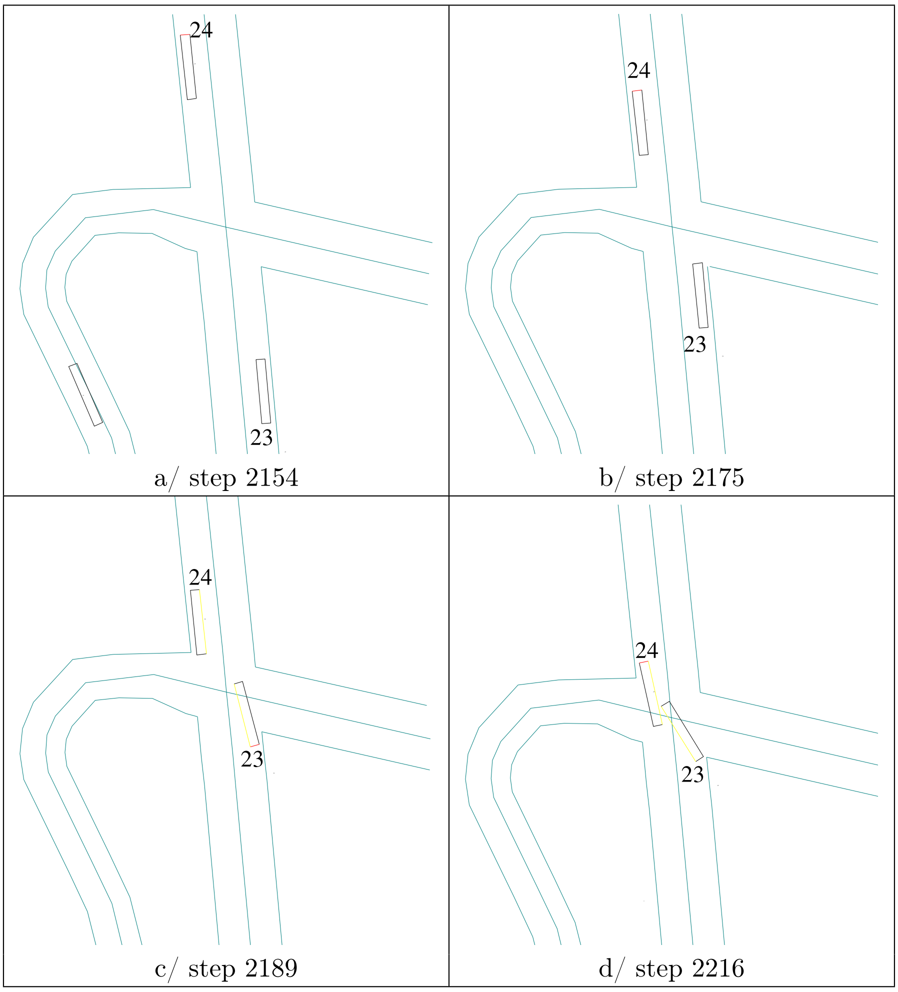

Figure 10: Simulation sequences without anticipation in ArchiSim

Figure 11 presents the same situation but this time the behavior of the bus is different because the agent is endowed with anticipatory abilities. At step 2175 (figure 10.b), bus 24 decides to enter the intersection. At step 2189 (figure 10.c), bus 23 has also reached the limit of the intersection, but before entering the intersection, the agent considers the situation. It constructs a simple mental representation of the situation involving bus 24 and itself. During this time, bus 24 continues on its trajectory, creating an anticipatory blocking relationships between 24 and 23. By calculating its future position (if it chooses to enter), bus 23 determines that it will be on the same trajectory as bus 24. The agent's mental representation has two variables (i.e., bus 24 and itself), and these two variables are linked by two blocking relationships: $bpr_{23}(23, 24)$ and $bpha_{23}(24, 23)$. The two domains are both initialized with $[1, 20]$ (which mean that agent 23 is able to anticipate before 20 seconds has passed). The constraint propagation on this simple network reduces the domains of 23 and 24 as described below:

- in the first iteration, propagating $bpr_{23}(23, 24)$ means that $dom(23) = [1, 20] - [1, 12] = [12, 20]$, where 12 is the time in seconds needed for 24 to move past the conflict point;
- in the same iteration, propagating $bpha_{23}(24, 23)$ means that $dom(24) = [1, 10] \cup [13, 20]$, where 10 is the time in seconds needed by 24 to reach 23;

- as $dom(24)$ has already been reduced, this modification demands a new propagation of $bpr_{23}(23, 24)$. Since 24 will be blocked between $t+11$ and $t+12$, 16 seconds are needed to move past the conflict point instead of 12 seconds and thus $dom(23)$ is now equal to $[16, 20]$;
- this modification of $dom(23)$ calls for a new propagation of $bpha_{23}(24, 23)$, which leads to a new reduction of $dom(24)$, and so on;
- in the end, two domains are obtained: $dom(23) = \varnothing$, $dom(24) = [1, 10]$.

For agent 23 having a empty domain is an undesirable state since it corresponds to infinite gridlock. As a result, agent 23, chooses to not enter the intersection but rather to stop (step 2247, figure 10.d). The same reasoning can be applied by agent 24. Since agent 23 has decided to stop, agent 24 detects the nil speed of 23 and does not add any *bpha* relation to its mental representation. Its anticipatory reasoning does not detect any gridlock; consequently it chooses to move.

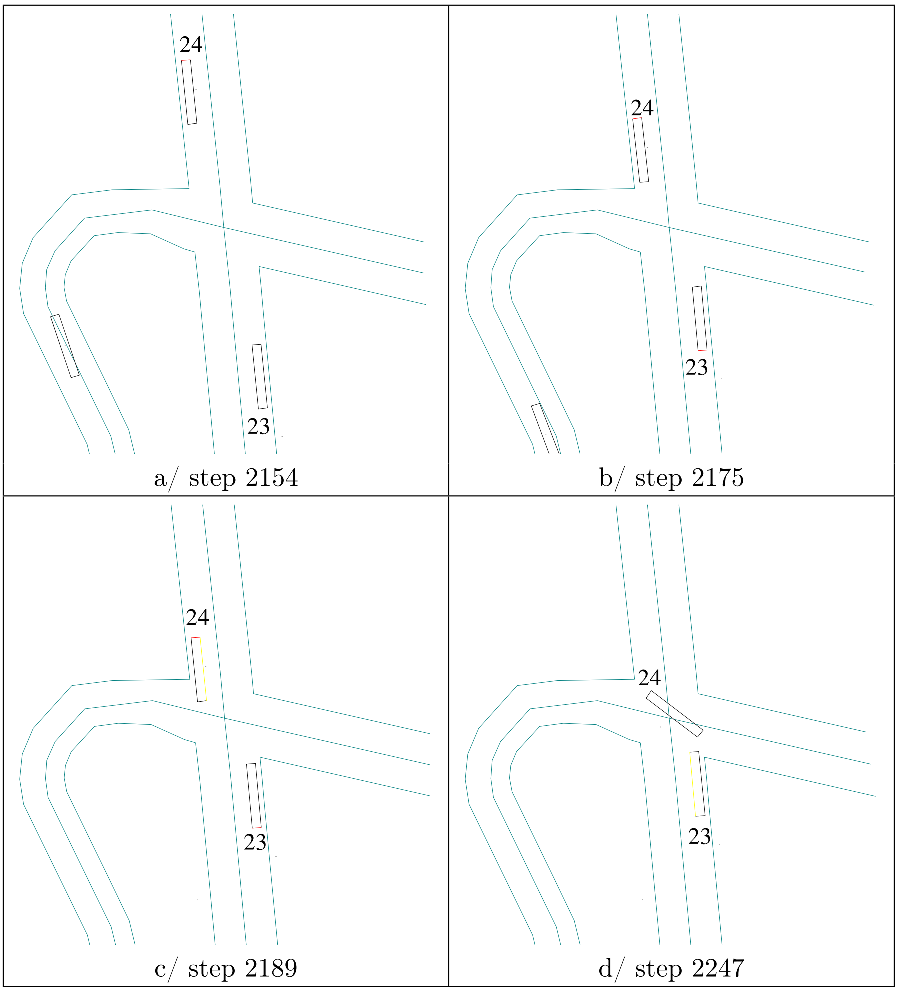

Figure 11: Simulation sequences with anticipation in ArchiSim

4.2 The simulation of a real intersection

The aim of this section is to show that our proposition is able to model driver behavior in a variety of conditions. Specifically, we try to show that our model is sufficiently generic to model different types of drivers evolving in different types of intersections. In our evaluations, we consider an Italian intersection and an American roundabout.

4.2.1 Comparison with real data for an italian intersection

The first intersection used for this evaluation is located in the city of Reggio Calabria in Italy. This intersection is the junction between a main road going from north to south and a secondary street going from east to west. Two stop signs are situated at the point where the secondary street meets the main road (figure 12). Each branch of the intersection has two lanes and the inner core of the intersection is large enough to allow vehicles to sit and wait. A 3D representation of this intersection is shown in figure 3. Since the driving rules are similar in many countries, particularly in France and in Italy, our explanations remain valid. The essential differences are to be found in the behavior of the drivers. For example, when arriving at a Stop sign, the average waiting time is around 3 seconds in France, whereas it is virtually zero in Italy.

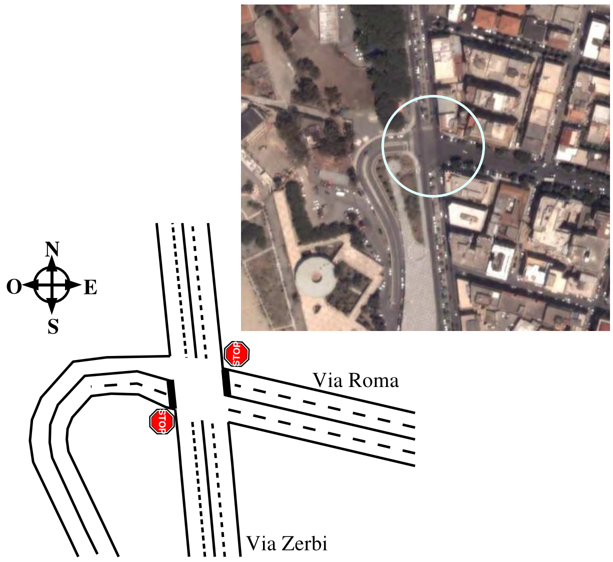

Figure 12: Description of the infrastructure of a real intersection (Reggio Calabria, southern Italia)

The University of Reggio Calabria manually measured the vehicle traffic at the intersection on a normal weekday between 12:30 PM and 1:30 PM, a rich period in terms of traffic density. For each branch of the intersection, the entry flow was expressed as a number of vehicles per hour and percentage of vehicles making right or left turns or proceeding straight ahead. Based on these data, we generated a traffic demand, which was used by ArchiSim to create simulated vehicles two hundred meters prior to the intersection. Virtual sensors were used to measure the flow of vehicles.

Simulations were carried out for two experimental conditions. For the first condition, agents used the basic behavior (Champion et al. (2003)), whereas our behavioral approach was used for the second condition.

Figures 13 to 16 compare the real flow measured with the simulated flows obtained with or without anticipation. In all the graphs shown, time is expressed in five minute' intervals.

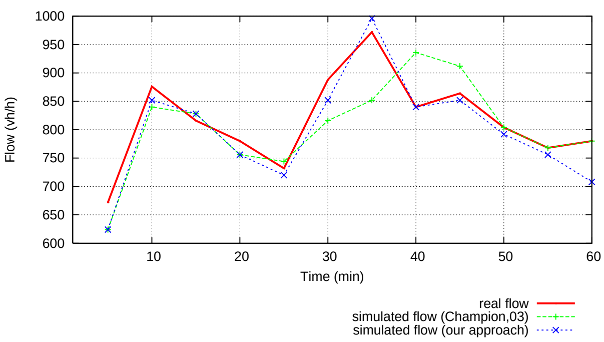

Figure 13: Simulated flow (South-North axis)

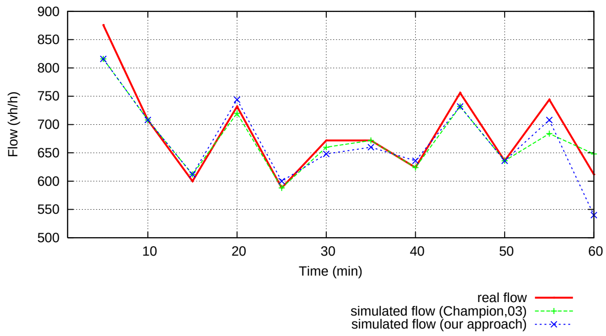

Figure 14: Simulated flow (North-South axis)

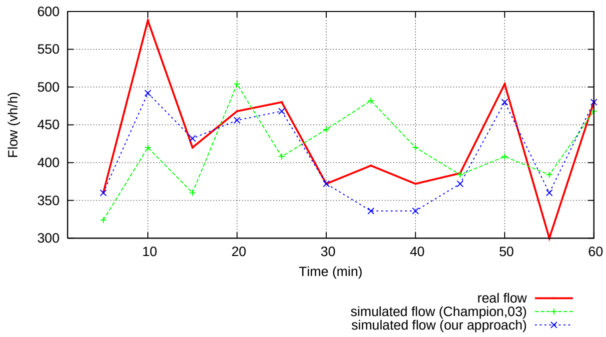

Figure 15: Simulated flow (East-West axis)

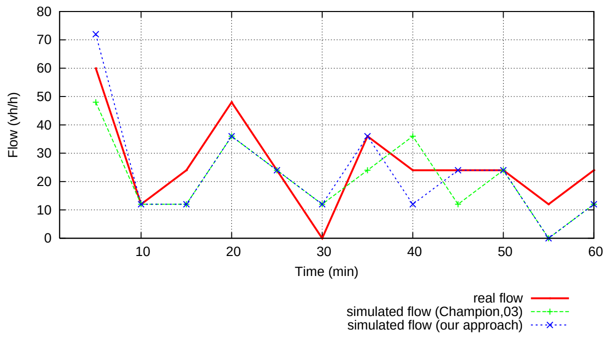

Figure 16: Simulated flow (West-East axis)

Figure 13 represents the flow curve for the principal south-north axis. According to this graph, introducing opportunistic behavior improves the validity of the flow, particularly in simulation. Indeed, around the 30th minute, the density of the traffic flows obtained based on the basic behavior decreases compared to the real flow: 820 vehicles instead of 900. This decrease is amplified 5 minutes later, leading to a variation compared to the real flow of 125 vehicles/hour. The simulation based on the basic behavior reaches its maximum in terms of traffic flow around the 40th minute, whereas in reality the peak was reached 5 minutes earlier. This phenomenon was not observed in the second simulation, in which the traffic flow reached its maximum at the 35th minute, just as the real situation.

Figure 14 represents the flow curve for the principal north-south axis. The results on this axis for the basic behavior are good, with a simulated curve similar to the one for the real flow. The results remain good following the introduction of our opportunistic behavior model.

Figure 15 represents the flow curve for the secondary east-west axis. This axis has a Stop sign. The graph in this figure shows that the first simulation, based on the basic behavior, is quite different from the real flow. At the 10th minute, a peak in the flow was observed at 600 vehicles/hour. In the simulation based on the basic behavior, the flow observed during the same period falls to 420. The second simulation greatly improves the traffic flows by reducing the difference. Globally, the shape of the curve for the simulated flows is identical to the one for the real flows. The difference at the 10th minute decreases from 180 to 100 vehicles/hour.

Figure 16 represents the curve for traffic flow on the secondary west-east axis. These results do not appear to be significant, given the low density of the flows on this branch of the intersection. The maximum flow is 60 vehicles/hour, which is quite low compared to the flow on the other three branches of the intersection.

In all cases, the best results were obtained by using our opportunistic and anticipatory behavior. We used RSD (Root Standard Deviation), a well-known indicator for traffic simulations, to more precisely qualify our results. RSD is the square error between the real flow y_i and the simulated flow x_i and is expressed by the following formula: $\sqrt{\frac{\sum_i (x_i - y_i)^2}{\sum_i {y_i}^2}}$.

For three of the four axis, the values obtained are less than 0.1, which corresponds to an error of less than 10% (table 1). Given that traffic engineers consider a traffic simulation to be good quality if its error rate is less than 15%, these results appear to be quite good. The results depicted on the curve in figure 16 are not very representative since the simulated flows were quit light and thus not very significant.

Table 1: Variation in RSD with basic and opportunistic/anticipatory behavior

	RSD		Flow (vh/h)	
	Champion et al. (2003)	our approach	Min	Max
North/South	0.06	0.04	600	900
South/North	0.03	0.03	680	980
East/West	0.15	0.06	300	600
West/East	0.32	0.30	0	60

4.2.2 Comparison with real data for an American roundabout

The second intersection considered for our evaluation is a roundabout located in the USA. Traffic data for this roundabout have been reported by Bared and Edara (2005). In this article, the authors evaluate the capacity with the traffic simulation tool, Vissim (Vissim (2005)). In their experiments, they compare real and simulated entry flows for different values of conflicting flows (i.e., the vehicle flow on the inner part of the roundabout). The simulation results obtained by the authors conform to reality: the higher the level of conflicting flows,the lower the entry flow.

Using the data presented in the paper cited in the previous paragraph, we conducted the same experiments using ArchiSim. Figure 17 gives an overview of the infrastructure and the geometry of the roundabout.

In our experiments, virtual sensors were set on the roads N1 and N6 in order to measure the flow inside (i.e., the conflicting flow) and at the entrance of the roundabout. Vehicles were created on the roads N5 and N6 (about 200 meters before the roundabout) with a desired speed varying between 48 km/h and 58 km/h (these values are given by Bared and Edara (2005)). The distribution of the desired speeds follows a Gaussian law around 53 km/h with a standard deviation inferior or equal to 5 km/h. The flow on N1 varies every ten minutes and grows from 120 to 900 vehicles per hour. The results obtained with ArchiSim are presented in figure 18 and in table 2 using the RSD indicator. In the figure 18, the x-coordinate represents the flow inside the roundabout and the y-coordinate, the entry flow. The curve obtained with ArchiSim shows that the simulated flow is more realistic than the one obtained with Vissim. The same conclusion can be made with respect to the RSD indicator (table 2).

4.2.3 The reproductibility of the simulation

In order to be used by traffic engineers, a traffic simulation tool has to be robust and must be able to simulate identical traffic phenomena over many consecutive executions. Traffic simulation based on

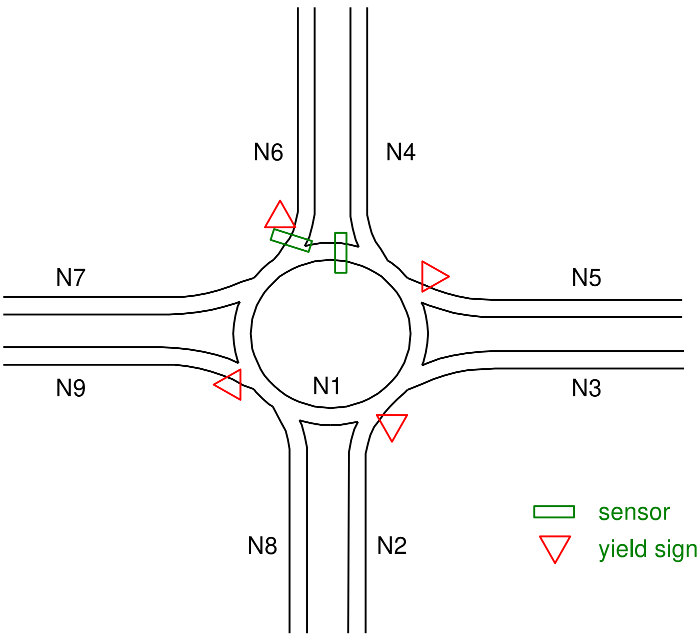

Figure 17: Geometry of the modeled roundabout

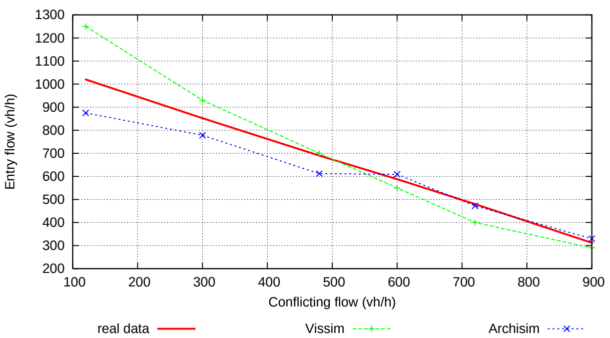

Figure 18: Flow comparison with ArchiSim and Vissim

Table 2: Variation in RSD between ArchiSim and Vissim

Conflictual Flow	RSD	
	ArchiSim	Vissim
120	0.14	0.23
300	0.09	0.09
480	0.11	0.02
600	0.04	0.06
720	0.02	0.17
900	0.06	0.07

mathematical approaches has no problem fullfilling this requirement since an equation always gives the same result. However, in multi-agent simulations, the large number of inter-agent interactions makes it difficult to control emergent phenomena. Consequently, the reproductibility of a particular situation is not always ensured.

For these reasons, in addition to presenting the good results obtained for the traffic flows, we thought it relevant to analyze the reproductibility of the traffic phenomena when using our approach. Specifically, we evaluated the stability of the traffic simulation for the data from the Italian intersection by varying the length of the time step. Because ArchiSim allows the duration of a simulation step to be parametrized, the simulations can be carried out with a fixed step varying between 1 and 30 milliseconds. Simulations can also be done in real time, which is useful when linking a driving simulator to the traffic simulation: 1 second of computing time is thus equivalent to 1 second of simulation.

The results reported in Table 2, obtained with a time step configured to run the simulation in real time, were verified at different fixed time steps: 1 ms, 5 ms, 10 ms and 20 ms. The comparison of the simulations reproduces the RSD variation. In general, there were very few changes in the traffic phenomena observed during the one-hour simulation. All of the simulations produced very good results in terms of RSD. These results are shown in the graph in figure 19. For each branch of the intersection, the RSD variation observed for the different simulations remains slight. On average, the RSD value varies from 0.009 to 0.02. The strongest variation is on the east-west axis, where the difference reaches to 0.02, confirming that the insertion of vehicles from a nonpriority traffic flow is a sensitive phenomenon. Globally, it appears that the simulation is reproductible with different time steps.

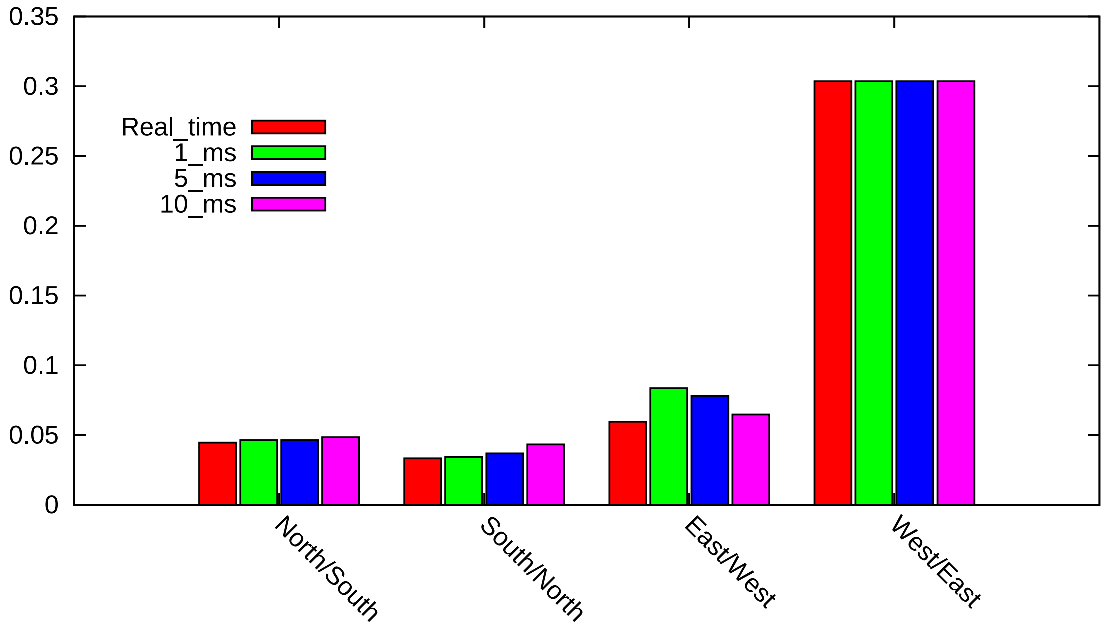

Figure 19: Variation of the RSD for different time steps

5 Conclusion and future works

In this paper, we have presented an approach to simulating traffic, based on traffic phenomena emerging from the local behaviors of simulated drivers. Because traffic simulation in intersections, which is not handled very well in most of the current commercial tools, we decided to consider this problem as a multi-agent coordination issue. In our approach, each simulated vehicle attempts to coordinate its actions with those of the other vehicles present in the intersection, or approaching it, in order to solve the potential conflicts.

The proposed model has two parts.

- The first part focuses on the perceptions of the surrounding traffic situation. We have shown that this perception of the situation influences agent's decision-making and thus the behaviors generated during the simulation. Within this framework, we introduced opportunistic behaviors that permit the simulated drivers to choose when to respect the Highway Code rules.
- The second part of our model aims to give anticipation abilities to the simulated drivers, introducing an anticipation algorithm that allows the agents to reason about their actions and thus avoid gridlock situations as much as possible.

This model was validated statistically by simulating real intersections and comparing the real and the simulated traffic flows. This comparison confirmed the relevance of our approach and validated the emergence of traffic phenomena from local agent behaviors.

Research is currently under way to apply ArchiSim to deal with the movements of motorbikes in road traffic. The central focus is to determine how motorbike drivers redefine their driving surface in terms of the vehicles already present on the roadway. To complete the model, we plan to manage the pertubations (e.g., traffic backup) that result from several consecutive intersections.

6 Acknowledgments

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